STUDIES ON OPTIMIZATION OF LAMINATED COMPOSITE PLATES

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STUDIES ON OPTIMIZATION OF LAMINATED COMPOSITE PLATES

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CERTIFICATE

This is to certify that the work STUDIES ON OPTIMIZATION OF LAMINATED COMPOSITE PLATES has been carried out under my supervision and has not been submitted elsewhere for a degree.

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LIST OF SYMBOLS

a, b	: Dimensions of the plate
A_{ij}, B_{ij}, D_{ij}	: Elastic constants of the composite plate
N_x, N_y	: In-plane normal forces
N^*_x	: Lower limit on Buckling load per unit width
K	: Ratio of in-plane normal forces (N_y/N_x)
$\bar{Q}_{ij}$	: Reduced stiffnesses evaluated at fiber orientation angle.
q	: Lateral loading
t_i	: Thickness of i^{th} lamina
u, v	: In-plane displacements
w	: Lateral displacement
$\epsilon_x, \epsilon_y, \epsilon_{xy}$	: Strain component in the xy-plane
$\sigma_x, \sigma_y, \sigma_{xy}$	: In-plane stresses
i	: Fiber orientation of ith lamina
V_i	: Design variables
K'	: Total number of design variables
N	: The number of plies in laminate

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ABSTRACT

Studies are carried out for the minimum weight optimum design of laminated fiber composite plates, subjected to multiple inplane loadings and transverse loadings. The behaviour constraints are on stability, strength and deflection. Angle-ply laminates with orthotropic laminae and antisymmetric angle-ply laminates are considered for optimization studies. The orthotropic property is achieved by taking an equal number of fibers at positive and negative orientations, with respect to structural axis in the former case. Thickness of plies and corresponding fiber orientations are incorporated as design variables while the number of plies is treated as a design parameter. The constrained optimization problem is transformed into a series of unconstrained optimization problems, using an interior penality function approach. The results have been obtained for different aspect ratios and uniform biaxial in-plane loading ratios. This study shows that the fiber orientations

of thickness near mid-plane have little effect on the optimum design. There exists a particular fiber orientation angle for the overall thickness of laminate, which results in the optimum design for a plate of a given aspect ratio under a given set of loadings. In case of antisymmetric angle-ply laminates, particular ratios of ply thickness result in the optimum design of the plate. It has been observed that the number of plies have no effect on the optimum design.

CHAPTER 1

INTRODUCTION AND LITERATURE REVIEW

1.1 INTRODUCTION

During the last two decades or so, fiber reinforced laminates have found increasing applications in many engineering structures. They are largely used in aeronautical industries for both primary and secondary structures. The fiber reinforced composites possess two desirable features: one is their high stiffness-to-weight and strength to weight ratio, and the other is their anisotropic material property that can be tailored through variation of the fiber orientation thickness of plies and stacking sequence - a feature which gives the designer an added degree of flexibility to achieve the desired strength or stiffness in any direction. The everincreasing use of composites have stimulated interest in the development of optimal design of structures made of composite materials.

Laminated plates constitute an important structural element in aerospace, mechanical and electrical industries. Laminated fibrous composites are a class of composites involving both fibrous composites and lamination techniques. Here, layers of fiber reinforced material are built up with the fiber direction of each layer typically oriented in different direction, to give different strengths and stiffnesses in the various directions, as desired by the

designer. Examples of laminated fiber reinforced composites include aerospace vehicles' outer covering, missile cases, aircraft wing panels and body sections, rocket motors, heat shields, fiber-glass boat hulls, tennis rackets, golf club shafts, etc..

In aerospace applications, where weight saving is of paramount importance, the advent of high modulus/high-strength fiber reinforced composite materials, such as carbon/epoxy and graphite/epoxy, has resulted in a dynamic increase in the use of laminated fiber reinforced plates and other structural shapes as primary structural members. The past, present and future of the composites in aircraft and space vehicles can be summarized as follows: The advanced composites were developed around 1960 and then the conceptual and structural feasibilities were demonstrated. Up to 1975, small components of hardware had been developed and flight tested. At present, large primary structures are being investigated. The latest journals reveal that NASA plans to build air-frames by using up to 80 percent composite materials. The final stage is the all-composite airplanes which will be seen in the near future. To achieve this, presently, optimization techniques have been applied to get an optimum weight design with desirable structural properties.

In the next section, a survey of the existing literature pertaining to optimization is presented.

1.2 LITERATURE SURVEY

Optimization in composite structural design was recognised as early as 1970. References [1-3] deal with optimum use of reinforcement using stiffness or strength as a design criterion.

Khot et. al. [4] suggested an efficient optimization method based on strain energy distribution and a numerical search for the minimum weight design of structures made from fiber reinforced composites. The optimum design procedure takes into consideration multiple loading conditions and displacement constraints on the structure. Sample problems consisting of both isotropic and composite elements are solved and the results presented.

Lai and Achenbach [5] used a direct search procedure to optimize layered structures, subjected to dynamic loading conditions. The technique is applied to problems of optimal design for minimum tensile stress at the interfaces in layered structure,under time harmonic loads and transient loads respectively.

Schmit and Farshi [6] presented a method for minimum weight design of symmetric composite laminates subjected to multiple in-plane loading conditions,which take into account membrane stiffness requirements and

strength limitations. The problem is cast as a nonlinear mathematical programming problem in which the thickness of material is a design variable with preassigned orientation angles.

Taisakson et. al. [7] applied optimality criteria in the iterative re-sizing of the elements of finite element model to achieve minimum weight. The program nominally treats a single generalized deflection constraint, but multiple constraints can be accommodated in many cases of practical interest by multiple submissions of the program.

Khot [8] wrote a computer program (OPTCOMP) which can be used to optimise or analyse a composite structure. Response of the structure to the applied loads is obtained by finite element analysis. The design variables are modified during each iteration by using a recurrence relation. The four strength criteria included in the program are: maximum stress, maximum strain, Hill's criterion modified by Tsai, and Norris criterion. The plate elements can be designed to prevent local buckling.

McKeown [9] presented an algorithm based on the simplex method of linear programming, without linearizing the non-linear mixed integer programming problem involving the number of layers, their thickness and their fiber directions. Orthotropic layers were used and the assumptions of the

analysis require that the lay-up be symmetric about the mid plane of the composite. It is assumed that layers are available in such a range of thickness that this would always be possible.

McKeown [10,11] dealt with the problem of optimizing multilayer fiber reinforced continua. The formulation variables are the nodal deflections of the finite element model. It is shown that program is convenient for solving the nonlinear mixed integer program involved and allows the optimal number of layers in each element. The layer thickness and the fiber angles were determined simultaneously for very general inequality constraints on stress and deflections.

Chao et. al. [12] determined optimum orientation for a symmetric composite laminate consisting of layers of + ve and - ve fiber orientations and subjected to in-plane loadings. Stability of both the laminate and laminae yield are evaluated. The buckling results presented are restricted to specially orthotropic laminates while the first laminae yield is assumed to initiate laminate failure.

Schmit and Farshi [13] presented a method for the minimum weight optimum design of laminated fiber composite plates, subjected to multiple in-plane loading conditions,

which include stiffness, strength, and elastic stability constraints. The buckling analysis is based on an equivalent orthotropic plate approach. This approach is appropriate for heavily loaded, balanced, symmetric laminate involving relatively large number of plies. The optimization technique used is a sequence of linear programming technique with constraint deletion techniques in conjunction with Taylor series approximations for the constraints retained. Thickness of plies and number of plies are used as design variables while fiber orientation is preassigned.

Hayashi [14] presented optimum design of plates made of angle ply laminates. The fiber orientations are optimized for the corresponding buckling strength. However, he considers the thickness of the laminae to be preassigned.

Hirano [15] carried out the optimum design of laminated plates with orthotropic layers under uniaxial and biaxial compression. Orthotropy is achieved by using equal number of fibers with + θ and - θ orientation in the same ply. Laminated plate is made up of plies with equal thickness. Fiber orientation of each ply is treated as design variable while the number of plies is preassigned. Powells conjugate direction method is used to maximize the buckling stress.

Recently, Sharma et. al. [16] have shown that, with variable thickness of plies, the degrading effects of coupling can be eliminated. This results in an increase in the strength of the laminate.

The survey of the foregoing literature on the optimum design of laminated composite plates shows that, not all the design variables have been exploited to arrive at an optimal design. Either fiber orientation or thickness of a ply has been treated as a design variable. Thus it seems a systematic study of optimization of composite plates is required by taking into account all the possible design variables.

1.3 OBJECTIVE AND SCOPE OF THE PRESENT WORK

The design problem with composites involves the determination of thickness distributions and corresponding fiber orientations. Thickness for each fiber orientation can always be subdivided into integer number of plies with given fiber orientation (this may be necessary because of manufacturing limitations). The penalty paid by slight variation in thickness in order to get integer number for plies can be exploited, which any way occurs when one starts with preassigned thickness or constraints on the number of plies to be an integer. The thickness distribution and corresponding fiber orientation should be chosen.

in such a way, that an optimal solution is obtained for a desired objective like minimum weight. At the same time behavioural constraints, like plate stability, etc., and side constraints, which include manufacturing tolerances and available gauge size, should be satisfied. If the objective function or the design requirements are dependent on other design variables or on a complex analysis, the optimization problem becomes very difficult. This, in fact, has to be in the optimum design of laminated composite plates.

In the present work, an attempt is made to incorporate thickness of each ply and fiber orientation as design variables, while the number of plies is used as a design parameter. Two types of problems are studied:

(1) Optimum design of a layered composite plate of a given dimension to have a maximum buckling load.

(2) Minimum weight design of the laminate subjected to in-plane and transverse loading.

In both the problems analytical derivatives are obtained. Interior penalty function approach is used for sequential unconstrained optimization using McCormick's modification of the Fletcher - Powell method for unconstraint optimization with Golden section method for unidirectional search.

A study of the buckling under biaxial compression is presented in Chapter 2. Buckling load is maximized to understand the behavior of fiber orientations. The second problem in chapter 2 incorporates thickness of eachply in addition to the fiber orientation as design variables.

Chapter 3 deals with an optimum weight design of composite plate under in-plane and transverse loadings. Composite plate stacked with orthotropic laminae is considered. Each lamina's principal directions coincide with structural axes.

Antisymmetric angle-ply laminates are considered in chapter 4, for optimization studies with constraints on stress, deflection and buckling load.

Chapter 5 summarises the results of the investigation. A short note is added to indicate the extension of the present work.

CHAPTER 2

OPTIMAL DESIGN OF LAMINATED PLATES UNDER UNIFORM AXIAL COMPRESSION

2.1 INTRODUCTION :

Since the early seventies the use of anisotropic fiber reinforced composites as structural members has increased considerably [17]. This increase owes to the fact that the designer can take advantage of the anisotropic properties of the materials. The anisotropic or layered composite materials in the form of thin plates, subjected to inplane compressive loads, are of particular interest. The primary mode of failure of these plates is buckling under axial compression. By appling optimization techniques, one can get an optimum design and thereby use these materials quite efficiently.

The present study is an attempt to understand the behaviour of the plate with varying thickness of the lamina and corresponding fiber orientation. A composite laminated plate, with each ply (lamina) having orthotropic property, is employed. Orthotropy is achieved by using equal number of fibers at + ve and - ve with respect to structural axis. Thus, stretching and shear are uncoupled for each ply. The reduced stiffness coefficients, $\overline{Q}_{16}$ and $\overline{Q}_{26}$, are identically zero. In this case, principal material directions and structural directions coincide.

The study is carried out for the following :

(1) Optimization of a layered composite plate for maximum buckling load with constraint on each ply to be within a preassigned thickness.

(2) Optimization of a layered composite plate for maximum buckling load with constraint on the total thickness of laminate to be within a preassigned value.

2.2 OPTIMIZATION PROBLEM

The constrained optimization problem considered herein is to find a vector of design variables, v^*, that maximizes the buckling load, $N_x(v^*)$, of a plate, subject to the constraints

$$g_i\ (v) \geq 0 \qquad i = 1, m \tag{1}$$

It is assumed that at least one of the constraints of Eq. (1) is active at the optimum point, that is, $g_i\ (v) = 0$ for some i. This constrained optimization problem may be transformed into a series of unconstrained minimization problems by introducing a penalty function associated with the constraints. In the transformed problem, we have to find the maximum of a function $P(r,v)$, as r approaches zero, where

$$P(r,v) = N_x(v) - r \sum_{i=1}^{M} f_i(v) \tag{2}$$

$f_i(v)$ is defined here as

$$f_i(v) = \ln\,(g_i(v)) \text{ if } g_i(v) \geq 0 \qquad (3)$$

The term $r\,f_i(v)$ represents the penalty associated with the i^{th} constraint in an interior penalty function, in the sense that it is defined only if v lies in feasible design domain. The above transformed problem can be solved by a sequence of unconstrained optimization problem, by Fletcher and Powell method [18].

2.3 PROBLEM FORMULATION

For the present investigation the objective function is

$$\text{Max } K_x\,(\alpha_i,\ t_i) \qquad i = 1,N$$

such that

$$0 \leq \alpha_i \leq \pi/2\ ;\ t_i \geq 0$$

and

$$(1)\ t_i \leq t_i^* \qquad i = 1,N \qquad (4)$$

$$(2)\ \sum_{i=1}^{N} t_i \leq t^*$$

These constraints are suitably formulated to fit in the interior penalty function approach.

2.3.1 ANALYSIS

Fig. 1.1 (a) shows an orthotropic ply with equal number of $+\alpha$ and $-\alpha$ fiber orientations. It should be noted that $\bar{Q}_{16}$ and $\bar{Q}_{26}$ are identically zero.

The three governing equations along u,v and w are (19)

$$A_{11} u_{xx} + A_{66} u_{yy} + (A_{12} + A_{66}) v_{xy} - B_{11} w_{xxx} - (B_{12} + 2B_{66}) w_{xyy} = 0 \quad (5)$$

$$(A_{12} + A_{55}) u_{xy} + A_{66} v_{xx} + A_{22} v_{yy} - (B_{12} + 2B_{66}) w_{xxy} - B_{22} w_{yyy} = 0 \quad (6)$$

$$D_{11} w_{xxxx} + 2 (D_{12} - 2D_{66}) w_{xxyy} + D_{22} w_{yyyy} - B_{11} u_{xxx} - (B_{12} + 2B_{66}) u_{xyy} - (B_{12} + 2B_{66}) v_{xxy} - B_{22} v_{yyy} + N_x w_{xx} + N_y w_{yy} = 0 \quad (7)$$

where (see fig. 1.1(b))

$$A_{ij} = \sum_{k=1}^{N} (\bar{Q}_{ij})_k (z_k - z_{k-1})$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{N} (\bar{Q}_{ij})_k (z_k^2 - z_{k-1}^2) \quad (8)$$

$$D_{ij} = 1/3 \sum_{k=1}^{N} (\bar{Q}_{ij})_k (z_k^3 - z_{k-1}^3)$$

and

$$N_y = K N_x$$

Uptill now the problem is quite general and can be applied to any set of boundary conditions. Now onwards the analysis is restricted to simply supported boundary conditions, which are used for optimization studies. We choose u,v, and w in the following manner to satisfy simply supported boundary conditions.

$$u = \bar{u} \cos (m \pi x/a) \sin (n \pi y/b)$$

$$v = \bar{v} \sin (m \pi x/a) \cos (n \pi y/b) \qquad (9)$$

$$w = \bar{w} \sin (m \pi x/a) \sin (n \pi y/b)$$

Substituting these assumed functions (9) in the governing equations (5,6,7) , and further simplification results in

$$N_x = \frac{1}{\pi^2 a^2 \{ m^2 + n^2 (a/b)^2 \}} \left[T_{33} + \frac{2 T_{12} T_{23} T_{13} - T_{22} T_{13}^2 - T_{11} T_{23}^2}{T_{11} T_{22} - T_{12}^2} \right] \qquad (10)$$

where

$$T_{11} = A_{11} m^2 \pi^2 + A_{66} n^2 \pi^2 (a/b)^2$$

$$T_{12} = (A_{12} + A_{66}) m n \pi^2 (a/b)$$

$$T_{13} = B_{11} m^3 \pi^3 + (B_{12} + 2 B_{66}) m n^2 \pi^3 (a/b)^2$$

$$T_{22} = A_{66} m^2 \pi^2 + A_{22} n^2 \pi^2 (a/b)^2$$

$$T_{23} = (B_{12} + 2B_{66})\, m^2 n\, \pi^3\, (a/b) + B_{22}\, n^3\, \pi^3 (a/b)^3$$

$$T_{33} = D_{11}\, m^4\, \pi^4 + 2\,(D_{12} + 2\,D_{66})\, m^2 n^2\, \pi^4 (a/b)^2$$
$$+ D_{22}\, n^4\, \pi^4\, (a/b)^4$$

2.3.2 DERIVATIVE OF BUCKLING LOAD

The application of optimization algorithm requires the gradient of objective function and constraints. Numerical computation leads to two function evaluation at each design point which leads to higher computational time. Because of the complex nature of the function, truncation errors increase and hence produce erroneous gradients/decrease the effeciency of the algorithm. (which inserted above "decrease") Derivatives of critical axial load with respect to design variables, which include thickness of ply and corresponding orientations, are obtained as follows.

Let v_i be design variable $i = 1, N'$

For this case,

$$v_i = t_i \qquad i = 1,\ N'/2 \quad \text{and} \quad v_i = \alpha_i \quad i = N'/2+1, N'$$

Differentiation of Eq. (10) gives,

$$\frac{\partial N_x}{\partial v_i} = \frac{1}{\pi^2 a^2 \left\{ m^2 + Kn^2 (a/b)^2 \right\}} \left[\frac{\partial T_{33}}{\partial v_i} \right.$$

$$+ (T_{11} T_{22} - T_{12}^2)\ (2\ \frac{\partial T_{12}}{\partial v_i}\quad T_{23}\ T_{13} + 2\,\frac{\partial T_{23}}{\partial v_i}\, T_{12} T_{13}$$

$$+ 2 \frac{\partial T_{13}}{\partial v_i} T_{23} T_{12} - \frac{\partial T_{22}}{\partial v_i} T_{13}^2 - 2 \frac{\partial T_{13}}{\partial v_i} T_{13}$$

$$- \frac{\partial T_{11}}{\partial v_i} T_{23}^2 - 2 \frac{\partial T_{23}}{\partial v_i} T_{11} - T_{23}) - (2 T_{12} T_{23} T_{13}$$

$$- T_{22} T_{13}^2 - T_{11} T_{23}^2) (\frac{\partial T_{11}}{\partial v_i} T_{22} + \frac{\partial T_{22}}{\partial v_i} T_{11}$$

$$-2 \frac{\partial T_{12}}{\partial v_i} T_{12}) / (T_{11} T_{22} - T_{12}^2)^2 \tag{12}$$

where,

$$\frac{\partial T_{11}}{\partial v_i} = \frac{\partial A_{11}}{\partial v_i} m^2 \pi^2 - \frac{\partial A_{66}}{\partial v_i} n^2 \pi^2 (a/b)^2$$

$$\frac{\partial T_{12}}{\partial v_i} = (\frac{\partial A_{12}}{\partial v_i} + \frac{\partial A_{66}}{\partial v_i}) m n \pi^2 (a/b)$$

$$\frac{\partial T_{13}}{\partial v_i} = \frac{\partial B_{11}}{\partial v_i} m^3 \pi^3 + (\frac{\partial B_{12}}{\partial v_i} + 2 \frac{\partial B_{66}}{\partial v_i}) m n^2 \pi^3 (a/b)^2$$

$$\frac{\partial T_{22}}{\partial v_i} = \frac{\partial A_{66}}{\partial v_i} m^2 \pi^2 + \frac{\partial A_{22}}{\partial v_i} n^2 \pi^2 (a/b)^2 \tag{13}$$

$$\frac{\partial T_{23}}{\partial v_i} = (\frac{\partial B_{12}}{\partial v_i} + 2 \frac{\partial B_{66}}{\partial v_i}) m^2 n \pi^3 (a/b)$$

$$+ \frac{\partial B_{22}}{\partial v_i} n^3 \pi^3 (a/b)^3$$

$$\frac{\partial T_{33}}{\partial v_i} = \frac{\partial D_{11}}{\partial v_i} m^4 \pi^4 + 2 (\frac{\partial D_{12}}{\partial v_i} + 2 \frac{\partial D_{66}}{\partial v_i}) m^2 n^2 \pi^4$$

$$(a/b)^2 + \frac{\partial D_{22}}{\partial v_i} n^4 \pi^4 (a/b)^4$$

Gradients of A_{ij} , B_{ij} and D_{ij} of Eq. (13) have been referred at various stages in this thesis. These stiffness matrices are a function of, both fiber orientation and thickness of ply. Expressions for gradients of stiffness matrices are given in Appendix 1.

2.3.3 CONSTRAINTS AND THEIR GRADIENTS

Constraints and their derivatives for two optimization studies carried out are as follows :

Following constraints are imposed on the first problem

$$g_i = 1 - v_i/t_i^* \qquad i = 1, N'/2 \tag{14}$$

$$g_i = 1 - v_i/W/_2 \qquad i = N'/2 + 1 \text{ , } N' \tag{15}$$

and

$$g_{i-N'} = v_i \qquad i = 1,N' \tag{16}$$

Derivatives of equation 14,15 and 16 are as follows

$$\frac{\partial g_i}{\partial v_j} = - \delta_{ij} \; 1/t_i^* \quad \text{for } i = 1,N'/2 \quad j=1,N' \tag{17}$$

$$\frac{\partial g_i}{\partial v_j} = - \delta_{ij} \; 1/ \;/2 \quad \text{for } i=N'/2 - 1, \; N' \; j=1,N' \tag{18}$$

Following constraints are imposed on the second problem.

$$g_1 = 1 - (\sum_{i=1}^{N'/2} v_i) /t^* \tag{19}$$

$$g_i = 1 - \frac{v_{i+N'/2-1}}{\pi/2} \qquad i = 2, N'/2 + 1 \tag{20}$$

Derivatives of constraints are

$$\frac{\partial g_1}{\partial v_j} = -\frac{1}{t^*} \qquad j = 1, N'/2$$

$$= 0 \qquad j = N'/2 + 1, N' \tag{21}$$

$$\frac{\partial g_i}{\partial v_j} = -\delta_{kj} \; 1/\pi/2 \qquad \text{for } i = 2, N'/2 + 1$$

where, $k = i + N'/2 - 1 \qquad j = 1, N'$

2.4 RESULTS AND DISCUSSIONS

2.4.1 RESULTS

Optimization studies have been carried out on rectangular composite plates, taking into consideration number of plies, aspect ratio and biaxial loading ratio as parameters for Boron/Epoxy composite whose material properties are as follows:

$E_1 = 2.11 \times 10^4$ kg/mm^2

$E_2 = 2.11 \times 10^3$ kg/mm^2

$\nu_{12} = 0.3$

$G_{12} = 7.03 \times 10^2$ kg/mm^2

Numerical computations have been carried out on DEC 1090 computer. Results obtained for two sets of constraints considered are as follows :

2.4.1(d)

Table 2.1 shows typical results obtained by varying number of plies. This indicates that the number of plies has no effect on maximum buckling load. Laminated plate assumes orthotropic properties at optimum design point, by orienting fibers of all the plies in the direction which results in maximum buckling load. Never-the-less, computational results obtained by considering higher number of plies provided some useful design informations which are discussed later in this section.

The variation of maximum buckling load with aspect ratio is shown in Table 2.2. The optimum fiber orientation increases from 0° to 90° with increasing aspect ratio for uniaxial loading. However, for biaxial loading with $K = (N_y / N_x) = 0.5$, the variation in fiber orientation is approximately from 9° to 62° for a change in aspect ratio from 0.5 to 2.0.

Table 2.3 shows that the buckling load decreases with increasing biaxial loading ratio. The optimum fiber orientation remains same for a square plate. However, for rectangular plates it increases with biaxial loading ratio. Typical results for aspect ratio 2.0 are presented in this table.

2.4.1(b)

Table 2.4 presents the optimum fiber orientation and corresponding buckling load for laminated plate, whose total thickness is constant but thickness of each ply can vary. These results indicate that the thickness of plies whose orientations are near optimum during first unconstrained minimization, blow up, thus reducing other plies to a negligible thickness to satisfy the constraint of maximum total thickness. In general, thickness of outer most plies increase, except in cases where initial orientations of inner plies are very near to optimum, while fiber orientation of outer plies are far away.

The variation of buckling load with fiber orientation for orthotropic laminated plate is shown in Fig. 2.2. Aspect ratio of plate is 1.25 and compressive load is applied along x direction only. It is observed that for fiber orientations up to 50°, a half sine wave in x-direction and a half sine wave in y-direction is the primary mode of buckling, but for higher orientations primary mode of buckling is two half sine waves in x-direction and one half sine wave in y-direction. Fiber orientation of 50° gives maximum buckling load and the mode of buckling also changes at this orientation. Fig. 2.3 presents similar analysis for aspect ratio 0.5 and $N_y/N_x = 0.5$.

Fig. 2.4 presents variation in fiber orientation for each unconstrained optimization done during sequential optimization.

2.4.2 DISCUSSION OF RESULTS

Fiber orientation of 45° leads to maximum buckling load for a square plate under uniaxial compression (Table 2.1). That, this will be so can be explained in the following manner.

Consider Eq. (10), which is reproduced in a dimensionless form. From Eq. (11) and (23) it is inferred that T_{33} predominantly contributes to the buckling load. (For orthotropic case, where $K = 0$, only T_{33} contributes to the buckling load)

$$\phi = \frac{12\, N_x\, b^2}{\pi^2 t^3\, Q_{22}} = \frac{12\,(b/a)^2}{\pi^4 t^3\, Q_{22}\left\{m^2 + K\, n^2 (a/b)^2\right\}} \cdot \left[T_{33} + \frac{2\,T_{12}\,T_{23}\,T_{13} - T_{22}\,T_{13}^2 - T_{11}\,T_{23}^2}{T_{11}\,T_{22} - T_{12}^2}\right] \qquad (23)$$

where

$$T_{33} = \pi^4 \left[D_{11}\, m^4 + 2\,(D_{12} + 2\,D_{66})\, m^2 n^2 (a/b)^2 + D_{22}\, n^4\, (a/b)^4\right] \qquad (24)$$

The bending stiffness coefficient D_{11} decreases with increase in orientation from 0° to 90°. At 90°, D_{11} is

equal to D_{22} at 0°. D_{22} increases with orientation from 0° to 90°. D_{22} at 90° is equal to D_{11} at 0°. At 45°, D_{11} and D_{22} are equal. In view of this variation, one can easily verify that T_{33} is maximum if fiber orientations are at 45° (m = 1 and n = 1 is the primary mode of buckling). This substantiates the results obtained by optimization study.

It may be noted that D_{22} in the expression for T_{33} Eq. (26) is multiplied by $(a/b)^4$ and 2 $(D_{12} - 2 D_{66})$ is multiplied by $(a/b)^2$. If aspect ratio (a/b) is less than one, then contribution of D_{22} diminishes with decreasing aspect ratio and D_{11} tries to push optimum fiber orientation towards 0°. $2(D_{12} + 2 D_{66})$ and D_{22} try to hold it back. As a result the optimum fiber orientation is in between 0° and 45°. For aspect ratios greater than one D_{22} contributes prominently and hence tries to pull up the fiber orientation. This results in the optimum fiber orientation between 45° and 90°. The above argument holds good if primary mode of buckling is a half sine wave in x and y-directions respectively. The change in buckling mode, changes contribution of each term in T_{33} expression which results in decrease in fiber orientation angle from 90° to 45° for higher aspect ratios.

The reasons for changes in mode shape for an orthotropic laminated plate (all plies with same fiber orientation) are discussed below.

(a) with n = 1, the change in the mode shape occurs, when ,

$$N_x\,(n=1,\ m) = N_x\,(n=1,\ m+1)$$

(load applied in x-direction only)

This results in

$$\frac{1}{m^2}\left[D_{11}\, m^4 + 2(D_{12} + 2D_{66})\, m^2 (a/b)^2 + D_{22}(a/b)^4\right]$$

$$= \frac{1}{(m+1)^2}\left[D_{11}\,(m+1)^4 + 2\,(D_{12} + 2D_{66})(m+1)^2 (a/b)^2\right.$$

$$\left. + D_{22}\,(a/b)^4\right] \tag{25}$$

Further simplification results in

$$(a/b)^4 = \frac{(2m+1)}{\left[\frac{1}{m^2} - \frac{1}{(m+1)^2}\right]} \cdot \frac{D_{11}}{D_{22}} \tag{26}$$

Eq. (26) gives the aspect ratio at which mode shape along x-direction changes. It is evident from equation (26) that the aspect ratio at which mode shape changes, depends on the material properties. For a plate of given thickness Eq. (26) can further be simplified, as,

$$(a/b)^4 = \frac{(2m+1)}{\left[\frac{1}{m^2} - \frac{1}{(m+1)^2}\right]} \quad \frac{\bar{Q}_{11}}{\bar{Q}_{22}} \tag{27}$$

Let $\bar{Q}^*$ and $\bar{Q}_{22}^*$ represent the reduced stiffness coefficients at optimum fiber orientation, then

$$(a/b^*)^4 = \frac{(2m+1)}{\frac{1}{m^2} - \frac{1}{(m+1)^2}} \quad \frac{\bar{Q}_{11}^*}{\bar{Q}_{22}^*} \tag{28}$$

for m = 1

$$(a^*/b)^4 = 4 \quad \frac{\bar{Q}_{11}^*}{\bar{Q}_{22}^*} \tag{29}$$

In order to obtain the aspect ratio for change in mode in x-direction at optimum fiber orientation, expression for optimum fiber orientation is derived. The condition for optimality with respect to fiber orientation is

$$\frac{\partial N_x}{\partial \sigma_i} = 0 \qquad i = 1, N$$

Reducing Eq. (10) to an orthotropic case and its differentiation results in

$$D_{11}' m^4 + 2 (D_{12}' + 2 D_{66}') m^2 n^2 (a/b)^2 + D_{22}' n^4 (a/b)^4 = 0 \tag{40}$$

where D_{ij}' are derivatives of bending stiffness coefficients with respect to fiber orientation, defined as below.

$$D_{ij}' = 1/3 \sum_{k=1}^{N} (\bar{Q}_{ij}')_k (z_k^3 - z_{k-1}^3) = \frac{t^3}{12} \bar{Q}_{ij}' \tag{41}$$

where,

$$\bar{Q}'_{11} = -2\,U_2 \sin 2\alpha - 4\,U_3 \sin 4\alpha$$

$$\bar{Q}'_{12} = 4\,U_3 \sin 4\alpha$$

$$\bar{Q}'_{22} = 2\,U_2 \sin 2\alpha - 4\,U_3 \sin 4\alpha \qquad (42)$$

$$\bar{Q}'_{66} = 4\,U_3 \sin 4\alpha$$

U_2 and U_3 are defined as

$$U_2 = \frac{Q_{11} - Q_{22}}{2}$$

$$U_3 = \frac{Q_{11} + Q_{22} - Q_{12} - 4Q_{66}}{8} \qquad (43)$$

Substituting Eq. (42) in Eq. (40) and after simplification it yields,

$$(U_2 \sin 2\alpha^* - 2\,U_3 \sin 4\alpha^*)\,(a/b)^4 + 12\,U_3 \sin 4\alpha^*$$

$$(a/b)^2 - (U_2 \sin 2\alpha^* + 2\,U_3 \sin 4\alpha^*)(a/b)^4 = 0 \qquad (44)$$

Eq. (44) gives the value of optimum fiber orientation for a given aspect ratio. To verify qualitative arguments given earlier, substitute $a/b = 1$ in Eq. (44), it results in ,

$$\sin 4\alpha^* = 0$$

and hence for maximum $4\alpha^* = \pi$ or $\alpha^* = \pi/4$ (45)

For $a/b = 1.25$ (Fig. 2.2) solving Eq. (44) by trial and error procedure, the following is obtained,

$$\gamma^* = 50.5 \quad \text{(Table 2.2)}$$

Solving Eq. (27) for this optimum fiber orientation the aspect ratio turns out to be 1.25 for a change in mode shape along x-direction (m=1 to 2). It shows that change in mode shape occurs at optimum.

(b) n = 1, proceeding in a similar way, the following Equations are obtained,

$$(a/b)^2 = \frac{(2n+1)}{(4n^3 + 6n^2 + 4n+1)} \cdot \frac{2(D_{12} + 2D_{66})}{D_{22}} \tag{46}$$

or,

$$(a^*/b)^2 = \frac{(2n+1)}{4n^3 + 6n^2 + 4n - 1} \cdot \frac{2(\bar{D}^*_{12} + 2\bar{D}^*_{66})}{\bar{D}^*_{22}} \tag{47}$$

for n = 1

$$(a^*/b)^2 = \frac{2}{5} \frac{\bar{D}^*_{12} + 2\bar{D}^*_{66}}{\bar{D}^*_{22}} \tag{48}$$

For biaxial loading, the aspect ratio at which the mode of buckling changes, becomes a complex function of material stiffness and hence of fiber orientations. For n = 1, Eq. (49) defines aspect ratio for which mode of buckling changes from m to (m+1) and is given below by

$$(a/b)^2 = \frac{K D_{11}(4m^3 + 6m^2 + 4m + 1) \pm}{}$$

$$\sqrt{D_{11}^2 (4m^3 + 6m^2 + 4m + 1)^2 + 4m^2(2m^3 + 5m^2 + 4m-1)}$$

$$\frac{(2m + 1)\ (D_{22} - 2K\ (D_{12} - 2D_{66}))\ D_{11}}{2(m-1)\quad D_{22} - 2K\ (D_{12} + 2D_{66})} \qquad (49)$$

Fig. 2.3 shows that crossing of mode 1.2 and mode 1:1 occurs at an orientation θ=10° which gives the maximum buckling load. This is plotted for an aspect ratio 0.5 and K = 0.5.

Table 2.2 reveals that fiber orientation of plies near mid-plane do not contribute much to maximum buckling load. It has been shown earlier that T_{33} contributes predominantly to buckling load and it contains bending stiffness co-efficients. If $\bar{Q}_{ij}$ is same for all the plies then the outer-most ply contributes maximum to D_{ij}. $(z_k^3 - z_{k-1}^3)$ is maximum for outer ply due to which T_{33} is maximum, which gives the maximum buckling load. Derivatives with respect to fiber orientation are also in tune with above results. Fig. 2.4 shows that fiber orientation of outer plies reaches optimum first and fiber orientation of plies near mid-plane approaches optimum slowly. The latter may terminate before reaching optimum orientation without making much difference in maximum buckling load.

Results for optimum buckling load with varying thickness and fiber orientation are presented in Table 2.4. Derivatives of buckling load with respect to thickness and fiber orientation of each ply are such that, the thickness of the ply that reaches optimum or near optimum fiber orientation earlier, keeps on increasing. so, as to enable a maximum fraction of the plate thickness to achieve optimum orientation. Results for this case also verify that contribution of plate thickness near mid-plane is not remarkable bu the outer most fiber orientations are important.

2.5 CONCLUSIONS

From the above study the following conclusion can be drawn.

(1) Maximum buckling load is obtained when total thickness of the plate assumes the optimum fiber orientation for a given aspect ratio and biaxial loading ratio. However fiber orientation and corresponding thickness can be distributed in such a way that they are almost equivalent to the total thickness having optimum fiber orientation.

(2) Since the optimum is obtained when total thickness assumes an optimum orientation, optimization technique

can be applied to a two ply plate to obtain optimum orientation. The results obtained by doing so can be used to design a plate of given aspect ratio for similar loading by varying the total thickness of plate which can be stacked with plies of given thickness with optimum fiber orientation.

(3) The designer of a composite plate for maximum buckling load can work with relax manufacturing tolerances as far as plies near the mid-plane are concerned.

(4) In general, optimum fiber orientation is such that, slight change in orientation changes primary mode of buckling : If a designer is interested in the mode of failure, he can choose fiber orientation slightly away from the optimum to ensure particular mode without any significant change in maximum buckling load. Any fiber orientation between 40° and 50° will result in almost maximum buckling load with associated mode 1:1 for a plate of aspect ratio 1.25 (Fig.2.2). Fiber orientation between 10 and 15 will result in maximum buckling load with associated mode 1:1 for a plate of aspect ratio 2.5 and biaxial loading ratio 0.5 (Fig. 2.3).

Table 4.1 EFFECT OF NUMBER OF PLIES ON MAXIMUM BUCKLING LOAD

No.of plies	Aspect ratio (a/b)	$K = \frac{N_y}{N_x}$	Optimum fiber orientation (degree) α_1	α_2	α_3	α_4	α_5	α_6	Reduced critical load ϕ	Mode of buckling
2	1.0	0.0	45.0	45.0					21.977	1:1
3	1.0	0.0	44.9	44.6	44.9				21.977	1:1
			45.0	45.0	45.0				21.977	1:1*
4	1.0	0.0	45.3	43.9	43.9	45.1			21.975	1:1
			45.0	45.0	45.0	45.0			21.977	1:1*
5	1.0	0.0	44.5	45.4	45.5	45.5	45.4	44.5	21.975	1:1
			45.0	45.0	45.0	45.0	45.0	45.0	21.977	1:1*

Table 3.2 VARIATION OF MAXIMUM BUCKLING LOAD WITH ASPECT RATIO

No. of plies	Aspect ratio (a/b)	$R = \frac{N_y}{N_x}$	Optimum fiber orientation (degree)						Reduced critical load $\emptyset$	Mode of buckling
			α_1	α_2	α_3	α_4	α_5	α_6		
6	0.5	0.0	3.02	5.20	15.50	15.50	4.20	1.8	42.02	1:1
			0.0	0.0	0.0	0.0	0.0	0.0	42.126	1:1*
	0.3	0.0	38.0	38.6	26.2	26.2	38.6	38.0	23.086	1:1
			38.6	38.0	38.0	38.0	38.0	38.0	23.130	1:1*
	1.0	0.0	44.5	45.4	45.5	45.5	45.5	45.5	21.975	1:1
			54.6	45.0	45.0	45.0	45.0	45.0	21.977	1:1*
	1.25	0.0	52.9	49.2	25.2	24.4	48.7	51.0	22.34	1:1 initial 20°
			49.8	50.2	55.4	56.1	49.0	50.2	23.38	2:1 initial 70°
			50.0	50.0	50.0	50.0	50.0	50.0	23.09	1:1*
			51.0	51.0	51.0	51.0	51.0	51.0	22.72	2:1*

TABLE 2.2 CONTINUED

No. of plies	Aspect ratio (a/b)	$K=\frac{K_y}{K_x}$	Optimum fiber orientation (degree) α_1	α_2	α_3	α_4	α_5	α_6	Reduced critical load ϕ	Mode of Buckling
6	2.0	0.0	44.9	45.0	44.9	44.9	45.0	44.9	21.977	2:1
			45.0	45.0	45.0	45.0	45.0	45.0	45.0	2:1*
	0.5	0.5	9.4	10.0	10.0	10.0	9.9	9.3	36.99	1:1
			8.3	4.0	22.56	1.16	7.13	11.95	36.98	1:1
			7.3	9.8	22.3	18.6	10.9	9.2	36.98	1:1
	1.0	0.5	45.2	45.3	48.2	51.8	44.0	45.2	14.64	1:1
			45.0	45.0	45.0	45.0	45.0	45.0	14.65	1:1*
	2.0	0.5	61.3	66.5	69.5	69.4	66.2	61.3	12.56	1:1 initial 70°
			67.6	51.6	26.9	22.7	49.9	68.9	12.25	1:1 initial 20°
			68.0	51.0	25.0	25.0	51.0	68.0	12.24	1:1°

TABLE 2.3 VARIATION OF MAXIMUM BUCKLING LOAD WITH BIAXIAL LOADING RATIO

No. of plies	Aspect ratio (a/b)	$K = \frac{N_y}{N_x}$	Optimum fiber orientation α_1	α_2	α_3	α_4	α_5	α_6	Reduced critical load ϕ	Mode of buckling
6	1.0	0.0	44.5	45.4	45.5	45.5	45.5	45.5	21.975	1:1
		0.5	45.2	45.1	48.2	51.8	44.0	45.2	14.64	1:1
	2.0	0.0	45.0	45.0	44.9	44.9	45.0	45.0	21.977	2:1
		0.5	67.3	51.0	27.0	22.7	50.0	68.9	12.25	2:1
		1.0	71.6	68.4	77.5	61.2	71.1	74.1	8.04	1:1
			74.0	69.0	55.0	56.0	65.0	76.0	8.00	2:1*
			75.0	75.0	75.0	75.0	75.0	75.0	7.58	2:1*

TABLE 2.4 OPTIMUM BUCKLING LOAD CONSIDERING THICKNESS AND FIBER ORIENTATION AS DESIGN VARIABLE

Number of plies 6

Aspect ratio	N_y/N_x	t_1/α_1	t_2/α_2	t_3/α_3	t_4/α_4	t_5/α_5	t_6/α_6	Reduced Buckling Load ϕ	Remark
		Optimum Values							
0.5	0.0	0.5	0.0	0.4	0.0	0.2	0.45	42.115	
		0.0	7.0	7.4	13.1	4.7	0.05		
		.254	.254	.254	.254	.254	.254		
		0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	
0.8	0.0	0.55	0.34	0.0	0.0	0.2	0.43	23.116	initial angle 30°
		38.6	34.1	31.1	31.0	33.9	38.9		
		0.59	0.17	0.0	0.0	0.21	0.55	23.069	initial angle 70°
		40.3	47.4	45.6	49.6	46.3	40.2		
		.254	.254	.254	.254	.254	.254	23.130	2.3 *
		38.0	38.0	38.0	38.0	38.0	38.0	[illegible]	

Results are less than [illegible] case

TABLE 2.4 CONTINUE

Aspect ratio	$\frac{N_y}{N_x}$	Optimum Values						Reduced Buckling Load ϕ	Remark
		t_1/α_1	t_2/α_2	t_3/α_3	t_4/α_4	t_5/α_5	t_6/α_6		
1.25	0.0	.255	.254	.253	.253	.254	.255	23.095	Mode 1:1
		50.1	50.0	50.0	50.0	50.0	50.1		
		.254	.254	.254	.254	.254	.254	22.72	*Mode 2:1
		51.0	51.0	51.0	51.0	51.0	51.0	23.116	
0.5	0.5	6.0	0.54	0.16	0.04	0.0	0.67	36.23	
		21.5	12.2	53.6	53.6	46.5	16.6		
		.35	.105	0.0	.49	0.0	.56	36.68	Mode 1:1
		12.0	15.3	22.2	19.5	14.9	11.1		
		.267	.261	.253	.243	.232	.267	37.00	
		9.5	9.7	9.9	10.0	9.8	9.4	37.024	

TABLE 2.4 CONTINUE

Aspect ratio	N_y/N_x	Optimum values						Reduced Buckling Load $\bar{\phi}$	Remark
		t_1/α_1	t_2/α_2	t_3/α_3	t_4/α_4	t_5/α_5	t_6/α_6		
2.0	0.5	.75	0.0	0.0	0.0	.01	.76	12.072	
		58.2	15.5	20.7	20.5	23.3	59.6		
		.08	.33	[illegible]	.35	.32	.08	12.509	
		65.6	62.0	60.3	60.3	62.0	65.6	12.556	
2.0	1.0	.54	.23	.03	.20	.14	.50	8.047	
		71.95	72.6	35.2	35.3	73.5	71.6		
		.87	0.0	0.0	0.0	0.0	.65	8.0457	
		72.0	12.8	27.5	21.3	20.1	71.6	[illegible]	

* Analysis

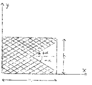

Fig. 1.1(a) - Co-ordinate System & fiber orientation

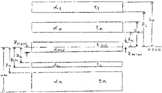

Fig.1.1 (b) - Stacking sequence & co-ordinate system for the layered plate.

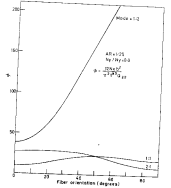

FIG.2.2 VARIATION OF BUCKLING LOAD WITH FIBER ORIENTATION

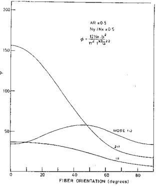

FIG. 2·3 VARIATION OF BUCKLING LOAD WITH FIBER ORIENTATION

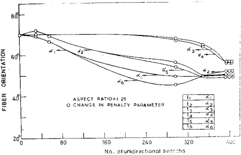

FIG. 2.4 CHANGE IN FIBER ORIENTATION DURING OPTIMIZATION

CHAPTER - 3

OPTIMUM WEIGHT DESIGN OF COMPOSITE PLATE UNDER IN-PLANE AND TRANSVERSE LOADING

3.1 INTRODUCTION

Minimum structural weight has been considered as an important creterion for structural design in aerospace engineering applications. The advantages of composites are that they exhibit higher strength to weight ratios than metals, and have better fatigue and corrosion properties. These desirable features have resulted in an increasing use of composites in aerospace structures. Weight saving by replacing metal by composites could lead to considerable fuel saving. Besides minimum weight requirement, aircraft structures are often desired for stiffness, strength, low deflection and high buckling loads.

In chapter 2, an optimum design of composite plates to withstand maximum in-plane compressive loads was discussed. In this chapter, an optimum weight design of composite plates is carried out. The composite plate is subjected to requirements on deflection under transverse loading, and high buckling strength under inplane compressive loads. The study is carried out for following cases :

(1) Optimization of weight of a composite plate under a low deflection requirement.

(2) Optimization of weight of composite plate under a high buckling load requirement and minimum deflection.

The first case is considered to understand the behaviour of deflection requirements under transverse loading. Behaviour of a plate under compressive loads is independently studied in chapter 2. Stresses in plies remain within ultimate value for all fiber orientations, and hence constraints on stresses are deleted during the optimization procedure, and an analysis is carried out at the optimum point.

3.2 PROBLEM FORMULATION

Weight per unit area per unit density of composite material is optimized to facilitate a parametric study.

$$\text{Min } W' \Rightarrow \text{Min } \sum_{i=1}^{N} t_i \qquad (50)$$

where $W' = \frac{W}{A}$

such that

$$0 \leqslant \alpha_i \leqslant \pi/2 \; ; \; t_i \geqslant 0 \qquad i=1,N$$

$$\omega \leqslant \omega^*$$

and

$$N_x \geqslant N_x^*$$

Constraints are suitably modified to fit into the interior penalty function formulation. An expression for buckling load is obtained in chapter 2 (Eq.(10)). In the following sections, expressions for maximum deflection and stresses are obtained.

3.3.1 ANALYSIS

The governing Equations (5,6 and 7) remain unchanged except that the term ($\bar{N}_x w_{xx} + \bar{N}_y w_{yy}$) is replaced by $-q$, the intensity of applied transverse load, in Eq. (7).

$$D_{11} w_{xxxx} + 2 (D_{12} + 2 D_{66}) w_{xxyy} + D_{22} w_{yyyy}$$

$$-B_{11} u_{xx} - (B_{12} + 2B_{66}) u_{xyy} - (B_{12} + 2B_{66}) v_{xxy}$$

$$- B_{22} v_{yyy} = q$$

The deflection of the simply supported plate is obtained by substituting Eq. (9) in Eqs. (5,6 and 7) accordingly.

$$\begin{vmatrix} -\frac{T_{11}}{a^2} & -\frac{T_{12}}{a^2} & \frac{T_{13}}{a^3} \\ \frac{T_{12}}{a^2} & -\frac{T_{22}}{a^2} & \frac{T_{23}}{a^3} \\ -\frac{T_{13}}{a^3} & -\frac{T_{23}}{a^3} & \frac{T_{33}}{a^4} \end{vmatrix} \begin{Bmatrix} \bar{u}_{mn} \\ \bar{v}_{mn} \\ \bar{w}_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ q_{mn} \end{Bmatrix} \quad (51)$$

where $q = \sum \sum q_{mn} \quad \sin \dfrac{m \pi x}{a} \ \sin \dfrac{n \pi y}{b}$

and T_{ij} are as defined in Eq. (11)

q_{mn} for uniform loading is,

$$q_{mn} = \frac{16\, q_o}{\pi^2} \ \frac{1}{mn} \qquad \begin{array}{l} m = 1, 3, 5 \ \ldots\ldots \\ n = 1, 3, 5 \ldots\ldots \end{array} \tag{52}$$

where

q_o - applied uniform loading per unit area

Eq. (51) is solved using Cramer's rule and following expressions are obtained.

$$\bar{u}_{mn} = \frac{q_{mn}}{a^5 \bar{D}} \left(T_{22}\, T_{13} - T_{12}\, T_{23} \right)$$

$$\tilde{u}_{mn} = \frac{q_{mn}}{a^5 \bar{D}} \left(T_{11}\, T_{23} - T_{13}\, T_{12} \right) \tag{53}$$

and

$$\bar{w}_{mn} = \frac{q_{mn}}{a^4 \bar{D}} \left(T_{22}\, T_{11} - T_{12}^2 \right)$$

where

$$\bar{D} = \frac{\left(T_{11}\, T_{22} - T_{12}^2 \right)}{a^8} \left[T_{33} + \frac{2\, T_{12}\, T_{13} T_{23} - T_{22}\, T_{13}^2 - T_{11}\, T_{23}^2}{T_{11}\, T_{22} - T_{12}^2} \right]$$

Knowing deformations, the maximum strains in a lamina can be obtained in the following manner

$$
\begin{aligned}
\epsilon_x &= u_x - z\, w_{xx} \\
\epsilon_y &= v_y - z\, w_{yy} \\
\epsilon_{xy} &= u_y + v_x - 2z\, w_{xy}
\end{aligned}
\tag{54}
$$

where u and v are the in-plane deflections at the selected surface, which is mid-surface in our case. Since the reference surface deflections u and v, the plate curvatures w_{xx} and w_{yy}, and the plate twist w_{xy} are all identical at any given point in the plate, the stresses at the surface z in the k^{th} lamina are given by

$$
\begin{Bmatrix} \sigma_x^{(k)} \\ \sigma_y^{(k)} \\ \sigma_{xy}^{(k)} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11}^{(k)} & \bar{Q}_{12}^{(k)} & \bar{Q}_{16}^{(k)} \\ \bar{Q}_{12}^{(k)} & \bar{Q}_{22}^{(k)} & \bar{Q}_{26}^{(k)} \\ \bar{Q}_{16}^{(k)} & \bar{Q}_{26}^{(k)} & \bar{Q}_{66}^{(k)} \end{bmatrix} \begin{Bmatrix} \epsilon_x^{(k)} \\ \epsilon_y^{(k)} \\ \epsilon_{xy}^{(k)} \end{Bmatrix} \tag{55}
$$

3.3.2 DERIVATIVES OF STRESSES AND TRANSVERSE DEFLECTION

The optimization algorithm requires the derivatives of stresses and transverse deflection. These are obtained as follows.

Let v_i be design variables $i = 1, N'$

For the present case,

$$v_i = t_i \qquad i = 1, N'/2$$

and

$$v_i = x_i \qquad i = N'/2 + 1, N'$$

Substituting Eq. (53) in Eq. (5) and differentiating, following gradients are obtained.

$$\frac{\partial u}{\partial v_i} = \frac{q_{mn}}{a^3} \left\{ \frac{D\left(\frac{\partial T_{11}}{\partial v_i} T_{23} + T_{11} \frac{\partial T_{23}}{\partial v_i} - \frac{\partial T_{13}}{\partial v_i} T_{12} - T_{13} \frac{\partial T_{12}}{\partial v_i}\right) - (T_{22} T_{13} - T_{12} T_{33}) \frac{\partial D}{\partial v_i}}{D^2} \right\}$$

$$\cos(m \pi x/a) \quad \sin(n \pi y/b) \tag{56}$$

$$\frac{\partial v}{\partial v_i} = \frac{q_{mn}}{a^3} \left\{ \frac{D\left(\frac{\partial T_{11}}{\partial v_i} T_{23} + T_{11} \frac{\partial T_{23}}{\partial v_i} - \frac{\partial T_{13}}{\partial v_i} T_{12} - \frac{\partial T_{12}}{\partial v_i} T_{13}\right) - (T_{11} T_{13} - T_{12} T_{23}) \frac{\partial D}{\partial v_i}}{D^2} \right\} \tag{57}$$

$$\sin(m \pi x/a) \quad \cos(n \pi y/b)$$

Let v_i be design variables $i = 1, N'$

For the present case,

$$v_i = t_i \qquad i = 1, N'/2$$

and

$$v_i = \alpha_i \qquad i = N'/2 + 1, N'$$

Substituting Eq. (53) in Eq. (9) and differentiating, following gradients are obtained.

$$\frac{\partial u}{\partial v_i} = \frac{q_{mn}}{a^3} \left\{ \frac{D\left(\frac{\partial T_{11}}{\partial v_i} T_{23} + T_{11} \frac{\partial T_{23}}{\partial v_i} - \frac{\partial T_{13}}{\partial v_i} T_{12} - T_{13} \frac{\partial T_{13}}{\partial v_i}\right) - (T_{22}\, T_{13} - T_{12} T_{33}) \frac{\partial D}{\partial v_i}}{D^2} \right\}$$

$$\cos (m \pi x/a) \quad \sin (n \pi y/b) \qquad (56)$$

$$\frac{\partial v}{\partial v_i} = \frac{q_{mn}}{a^3} \left\{ \frac{D \left(\frac{\partial T_{11}}{\partial v_i} T_{23} + T_{11} \frac{\partial T_{23}}{\partial v_i} - \frac{\partial T_{13}}{\partial v_i} T_{12} - \frac{\partial T_{12}}{\partial v_i} T_{13}\right) - (T_{11}\, T_{13} - T_{12} T_{23}) \frac{\partial D}{\partial v_i}}{D^2} \right\} \qquad (57)$$

$$\sin (m \pi x/a) \quad \cos (n \pi y/b)$$

$$\frac{\partial w}{\partial v_i} = \frac{T_{mn}}{a^4}\left[\frac{\delta \left(\frac{\partial T_{22}}{\partial v_i} + T_{11} + T_{22} \frac{\partial T_{11}}{\partial v_i} - 2\, T_{12} \frac{\partial T_{12}}{\partial v_i} \right)}{\delta^{\,2}} - (T_{22}\, T_{11} - T_{12}^2)\, \frac{\partial \delta}{\partial v_i} \right] \tag{58}$$

where

$$\frac{\partial \delta}{\partial v_i} = \frac{1}{a^8} \left(\frac{\partial T_{11}}{\partial v_i} T_{22} + T_{11} \frac{\partial T_{22}}{\partial v_i} - 2T_{12} \frac{\partial T_{12}}{\partial v_i} \right)$$

$$\Bigg[\frac{\partial T_{33}}{\partial v_i} + \Bigg\{ (T_{11}T_{22} - T_{12}^2)\Big(2 \frac{\partial T_{12}}{\partial v_i} T_{23}T_{13} + 2 \frac{\partial T_{12}}{\partial v_i}$$

$$T_{12}\, T_{13} + 2 \frac{\partial T_{13}}{\partial v_i} T_{23}\, T_{12} - \frac{\partial T_{22}}{\partial v_i} T_{13}^2 - 2 \frac{\partial T_{13}}{\partial v_i} T_{22}T_{13}$$

$$- \frac{\partial T_{11}}{\partial v_i} T_{23}^2 - 2 \frac{\partial T_{23}}{\partial v_i} T_{11} \Big) - \big(2\, T_{12}T_{23}T_{13} - T_{22}T_{13}^2 - T_{11}T_{23}^2\big)$$

$$\Big(\frac{\partial T_{11}}{\partial v_i} T_{22} + \frac{\partial T_{22}}{\partial v_i} T_{11} - 2 \frac{\partial T_{12}}{\partial v_i} T_{12} \Big) \Big/ (T_{11}\, T_{22} - T_{12}^2)^2 \Bigg\} \Bigg]$$

and

$\frac{\partial T_{ij}}{\partial v_k}$ are given by Eq. (13)

Gradients of stresses can be obtained in the following manner :

$$\frac{\partial \sigma_i^{(k)}}{\partial v_j} = k\frac{\partial \bar{Q}_{il}^{(k)}}{\partial v_j} + \bar{Q}_{il}^{(k)} \frac{\partial \epsilon_l}{\partial v_j} \qquad \begin{array}{l} i = 1, 2, \text{ and } 6 \\ l = 1, 2 \text{ and } 6 \\ j = 1, N' \end{array} \tag{60}$$

where

$$\frac{\partial \epsilon_x}{\partial v_j} = \frac{\partial u_x}{\partial v_j} - z \frac{\partial w_{xx}}{\partial v_j} - \frac{\partial z}{\partial v_j} w_{xx}$$

$$\frac{\partial \epsilon_y}{\partial v_j} = \frac{\partial v_y}{\partial v_j} - z\frac{\partial w_{yy}}{\partial v_j} - \frac{\partial z}{\partial v_j} w_{yy} \tag{61}$$

$$\frac{\partial \epsilon_{xy}}{\partial v_j} = \frac{\partial u_y}{\partial v_j} + \frac{\partial v_x}{\partial v_j} - 2z\frac{\partial w_{xy}}{\partial v_j} - 2\frac{\partial z}{\partial v_i} w_{xy}$$

Derivatives of $\{\bar{Q}_{il}\}$ and z are given in Appendix - 1 .

3.3.3 CONSTRAINTS AND THEIR GRADIENTS

Constraints and their derivatives for the optimization studies are obtained as follows.

The set of constraints described by Eqs. (14-16) in chapter 2 are also applied to minimum weight

optimization problem considered here. Their derivatives are given by Eqs. (17,18 and 22). In addition to these constraints, constraints on the maximum deflection and the ultimate stresses in principal directions or Tsai - Hill yield criterion for anisotropic materials, are applied in the first problem. Second problem includes constraint on buckling in addition to the constraints imposed on the first problem.

The additional constraints are

$$g_1 = 1 - w/w^* \qquad (62)$$

$$g_2 = (N_x/N_x^*) - 1 \qquad (63)$$

$$g_i = 1 - \frac{\sigma_{x\,i-2}}{\sigma_x^*} \quad i = 3,\ N+3 \qquad (64)$$

$$g_i = 1 - \frac{\sigma_{y\,i-(N+3)}}{\sigma_y^*} \quad i = N+4,\ 2N+4 \qquad (65)$$

$$g_i = 1 - \frac{\sigma_{xy\,i-(2N+4)}}{\sigma_{xy}^*} \quad i = 2N+5,\ 3N+5 \qquad (66)$$

or

$$g_i = 1 - \left(\frac{\sigma_{x\,i-2}}{\sigma_x^*}\right)^2 + \frac{\sigma_{x\,i-2}\,\sigma_{y\,i-2}}{\sigma_x^{*\,2}} - \left(\frac{\sigma_{y\,i-2}}{\sigma_y^*}\right)^2 - \left(\frac{\sigma_{xy\,i-2}}{\sigma_{xy}^*}\right)^2 \quad i = 3,\ N+3 \qquad (67)$$

where

$$\begin{Bmatrix} \sigma_x^* \\ \sigma_y^* \\ \sigma_{xy}^* \end{Bmatrix} = [T]^{-1} \begin{Bmatrix} \sigma_{Lu} \\ \sigma_{Tu} \\ \sigma_{LTu} \end{Bmatrix} \tag{58}$$

where

$$[T]^{-1} = [T]^T = \begin{vmatrix} \cos^2\alpha & \sin^2\alpha & -2\sin\alpha\cos\alpha \\ \sin^2\alpha & \cos^2\alpha & +2\sin\alpha\cos\alpha \\ 2\sin\alpha\cos\alpha & -2\sin\alpha\cos\alpha & (\cos^2\alpha - \sin^2\alpha) \end{vmatrix} \tag{69}$$

It should be noted that the fibers at orientations $+\alpha$ and $-\alpha$ will contribute to the ultimate strength in the principal directions, therefore,

$$\sigma_x^* = \sigma_{Lu} \sin^2\alpha + \sigma_{Tu} \cos^2\alpha$$

$$\sigma_y^* = \sigma_{Lu} \sin^2\alpha + \sigma_{Tu} \cos^2\alpha \tag{70}$$

$$\sigma_{xy}^* = \sigma_{LTu} (\cos^2\alpha - \sin^2\alpha)$$

Derivatives of constraints are

$$\frac{\partial g_1}{\partial v_i} = -\frac{1}{W^*} \frac{\partial W}{\partial v_i} \qquad i = 1, N' \tag{71}$$

$$\frac{\partial g_2}{\partial v_i} = \frac{1}{N_x^*} \frac{\partial N_x}{\partial v_i} \qquad i = 1, N' \tag{72}$$

$$\frac{\partial g_i}{\partial v_j} = -\frac{1}{\overset{*}{x}} \frac{\partial \sigma_{xi-2}}{\partial v_j} \qquad i=3,\ N+3 \quad j=1,N' \tag{73}$$

$$\frac{\partial g_i}{\partial v_j} = -\frac{1}{\overset{*}{y}} \frac{\partial \sigma_{yi-(N+3)}}{\partial v_j} \qquad i=N+4,\ 2N+4 \quad j=1,N' \tag{74}$$

$$\frac{\partial g_i}{\partial v_j} = -\frac{1}{\overset{*}{xy}} \frac{\partial \sigma_{xy\ i-(2N+4)}}{\partial v_j} \qquad i=2N+5,\ 3N+5 \quad j=1,N' \tag{75}$$

or

$$\frac{\partial g_i}{\partial v_j} = -2\,\frac{\sigma_{x\ i-2}}{x^{*2}} \frac{\partial \sigma_{xi-2}}{\partial v_j} + \frac{\sigma_{xi-2}}{\overset{*}{x}{}^{2}} \frac{\partial \sigma_{yi-2}}{\partial v_j} + \frac{\sigma_{yi-2}}{\sigma \overset{*}{x}{}^{2}} \frac{\partial \sigma_{xi-2}}{\partial \sigma v_j} - 2\,\frac{\sigma_{xyi-2}}{\sigma \overset{*}{x}y} \frac{\partial \sigma_{xyi-2}}{\partial v_j}$$

$$i = 3,\ N+3 \qquad j = 1,\ N' \tag{76}$$

3.4 RESULTS AND DISCUSSIONS

3.4.1 RESULTS

Minimum weight optimization of a rectangular plate has been carried out for varying number of plies and aspect ratios, for Boron/epoxy composite, whose material properties are as given in section (2.4.1).

$$\frac{\partial g_i}{\partial v_j} = - \frac{1}{\overset{*}{x}} \frac{\partial \sigma_{xi-2}}{\partial v_j} \qquad i=3,\ N+3 \quad j=1,N' \qquad (73)$$

$$\frac{\partial g_i}{\partial v_j} = - \frac{1}{\overset{*}{y}} \frac{\partial \sigma_{yi-(N+3)}}{\partial v_j} \qquad i=N+4,\ 2N+4 \quad j=1,N' \qquad (74)$$

$$\frac{\partial g_i}{\partial v_j} = - \frac{1}{\overset{*}{xy}} \frac{\partial \sigma_{xy\ i-(2N+4)}}{\partial v_j} \qquad i=2N+5,\ 3N+5 \quad j=1,N' \qquad (75)$$

or

$$\frac{\partial g_i}{\partial v_j} = -2 \frac{\sigma_{x\ i-2}}{x^{*2}} \frac{\partial \sigma_{xi-2}}{\partial v_j} + \frac{\sigma_{xi-2}}{\overset{*}{x}^2} \frac{\partial \sigma_{yi-2}}{\partial v_j}$$

$$+ \frac{\sigma_{yi-2}}{\sigma \overset{*}{x}^2} \frac{\partial \sigma_{xi-2}}{\partial \sigma v_j} - 2 \frac{\sigma_{xyi-2}}{\sigma \overset{*}{xy}} \frac{\partial \sigma_{xyi-2}}{\partial v_j}$$

$$i = 3,\ N+3 \quad j = 1,\ N' \qquad (75)$$

3.4 RESULTS AND DISCUSSIONS

3.4.1 RESULTS

Minimum weight optimization of a rectangular plate has been carried out for varying number of plies and aspect ratios, for Boron/epoxy composite, whose material properties are as given in section(2.4.1).

Numerical results have been computed for the following set of constraints :

Longitudinal tensile strength = 140.57 kg/sq.mm.
Transverse tensile strength = 8.43 kg/sq.mm.
Longitudinal compressive strength=-281.13 kg/sq.mm.
Transverse compressive strength=-28.11 kg/sq.mm.
Shear strength =12.65kg./sq.mm.
Applied uniform transverse loading =0.0005 kg/sq.mm.
Upper limit on maximum deflection=2.00 mm.
Minimum buckling load in x-direction=5.00kg/mm.

Table 3.1 presents the optimum ply thickness and corresponding fiber orientation for a minimum weight design of a plate subjected to uniform transverse loading. The deflection of the plate is restricted to be within prescribed upper limit. Results for different aspect ratios are tabulated. For each aspect ratio, a plate with varying number of plies is considered.

The variation of the maximum deflection with fiber orientation is plotted in Fig. 3.1 . The minimum of the maximum deflection of the plate is obtained at

higher fiber orientations as aspect ratio increases. The change in maximum deflection is small for plates of low aspect ratios while for plates of higher aspect ratios, it increases rapidly.

Figs. (3.2,3.3 and 3.4) show the variation of stresses in positive fiber direction, transverse to fiber direction, and shear stress with fiber orientation, respectively.

The optimum ply thickness and corresponding fiber orientations for plates with varying aspect ratio and biaxial loading ratio are presented in Table 3.2. Optimum fiber orientation angle increases with an increase in biaxial loading ratio for lower aspect ratios. The change is small as aspect ratio approaches unity. For a square plate the fiber orientation angle remains constant (45°) with increase in biaxial loading ratio. The contribution of plate thickness near mid-plane is not considerable. In fact, the fiber orientation of the plies near mid-plane do not achieve an optimum orientation, but their effect is nullified, to an extent, by reducing the thickness to an almost negligible value.

Fig. 3.5 shows the variation of the optimum weight with aspect ratio for different biaxial loading ratios. The optimum weight is higher for higher biaxial loading ratios, but at higher aspect ratios it remains unchanged with varying biaxial loading ratio. The maximum deflection and buckling load are plotted against aspect ratio in Fig. 3.6. The figure shows the active constraint for a given aspect ratio and biaxial loading, and it also represents the value of the other constraint for the same.

Fig. 3.7 presents the optimum weight for a given biaxial loading, for plates with different aspect ratios. The optimum weight remains unchanged for higher aspect ratios at lower biaxial loading ratios.

3.4. DISCUSSION OF RESULTS

In practice, a designer would be interested in knowing how the response quantities vary with a change in the design variable. In chapter 2, an attempt was made to understand the effect of stability criterion on the optimum design of the plate. In this chapter, the deflection under transverse loading is dealt with separately, to facilitate the designer in manipulating the design variables, and, sometimes, to understand the behavior of the design variables near optimum value

to help in rounding off the variables to the nearest practical value. Figs (3.1-3.4) present the results of the analysis. Fig. 3.1 presents the variation of the maximum dimensionless deflection of composite plate with fiber orientation. Variation of stress in positive fiber direction, stress transverse to positive fiber direction, and shear stress (σ_{LT}), with fiber orientation are shown in fig. 3.2, fig. 3.3 and fig. 3.4, respectively.

The requirement on deflection is very stringent in all aerospace designs. Either deflection or buckling is the governing design criterionand the stresses never exceed the ultimate value if the design remains within the requirements for deflection and stability. Figs. (3.2-3.4) indicate this behavior.(Boron filaments have a high modulus and high strength and low density. Aircraft structures are often designed for stiffness and strength and boron fiber is particularly suitable for low deflection and high strength applications.) Because the constraint on stress or strength criterion is never active near the optimum, these constraints are deleted during optimization procedure and only analysis is done at the final optimum point.

Eq. (10) and Eq. (43) show that the factor

$$1/\,\pi^2 a^2 \left\{ m^2 + Kn^2 \,(a/b)^2 \right\}$$

appears in the expression for N_x, and $a^4 q_{mn}$ appears in the expression for the transverse deflection. It means that at lower aspect ratios, buckling will be the governing criterion, while at higher aspect ratios, the deflection will be an active constraint. With higher values of biaxial loading, the change-over from buckling to deflection as an active constraint will take place at still higher aspect ratios. Analysis, similar to that done in section (2.4.2) will substantiate the results obtained for the optimum fiber orientations in table 3.2. The reasons given for the variations of ply thickness, at the end of section (2.4.2), hold good for results presented in table 3.1 and table 3.2.

The optimum weight does not depend upon the biaxial loading ratio if the constraint on deflection is active. It increases with increase in biaxial loading ratio because the buckling load decreases with increasing biaxial loading ratio and, hence, the thickness of the laminate has to compensate for it to get the design to satisfy the stability requirement (fig. 3.7).

3.5 CONCLUSIONS

The following conclusions can be drawn.

(1) The weight per unit area of the plate increases with increasing aspect ratio. However, it seems to approach an asymptotic value with aspect ratio.

(2) Stability criterion is the active constraint at lower aspect ratios. The transverse deflection is the active constraint at higher aspect ratios.

(3) The change, from the stability criterion to the deflection criterion as an active constraint, takes place at higher aspect ratios with increase in biaxial loading.

(4) Stresses never become active at optimum point with the low deflection requirement on the laminate design. These constraints can be deleted to improve the efficiency of the algorithm.

Conclusions given at the end of chapter 2, in general, hold for the minimum weight design under transverse deflection and stability constraints, too.

TABLE 3.1 – THE OPTIMUM VALUES OF THE PLY THICKNESS AND CORRESPONDING FIBER ORIENTATION FOR A PLATE

(ONLY TRANSVERSE LOADING IS APPLIED)

Aspect ratio	No of plies	Optimum Values						Optimum weight
		t_1/α_1	t_2/α_2	t_3/α_3	t_4/α_4	t_5/α_5	t_6/α_6	[illegible]
0.5	2	0.97 0.21	0.97 0.21					1.94
	4	0.92 0.09	0.04 39.65	0.04 39.65	0.93 0.05			1.94
	6	0.83 5.10	0.11 35.12	0.05 43.86	0.03 45.69	0.10 31.89	0.82 5.28	1.95
[illegible]	2	1.69 37.2	1.69 37.2					3.39
	4	1.47 37.2	0.25 42.[illegible]	0.19 42.25	1.47 37.2			3.39
	6	1.55 37.44	[illegible] 41.1	[illegible] 44.35	0.75 45.35	0.69 41.15	1.58 37.44	3.39

TABLE 3.1 CONTINUS

Aspect ratio	No. of plies	Optimum values t_1/α_1	t_2/α_2	t_3/α_3	t_4/α_4	t_5/α_5	t_6/α_6	Optimum weight $w/\rho A$
1.0	2	2.00	2.00					4.01
		45.0	45.0					
	4	1.00	1.00	1.00	1.00			4.01
		45.0	44.99	44.99	45.0			
	6	45.0	45.0	45.0	45.0	45.0	45.0	4.01
		0.67	0.67	0.67	0.67	0.67	0.67	
1.25	2	2.29	2.23					
		52.37	52.38					
	4	1.56	0.72	0.72	1.56			4.56
		53.[illegible]	46.54	46.53	53.02			
	6	0.34	[illegible]	[illegible].57	0.54	0.23	[illegible]	4.57
		52.[illegible]	52.2[illegible]	46.56	46.56	52.1[illegible]	52.72	

TABLE 3.1 CONTINUE

Aspect ratio	No.of plies	Optimum values						Optimum weight W/A
		t_1/α_1	t_2/α_2	t_3/α_3	t_4/α_4	t_5/α_5	t_6/α_6	
2.0	2	2.52 89.79	2.37 89.79					4.89
	4	2.33 89.7	0.12 51.29	0.11 51.22	2.33 89.69			4.89
	6	2.17 89.12	0.15 63.29	0.12 47.69	0.12 47.72	0.15 63.30	2.17 89.12	4.89

TABLE 3.2 VARIATION OF OPTIMUM VALUES WITH ASPECT RATIO AND BIAXIAL LOADING RATIO

Aspect ratio	$K = \frac{N_y}{N_x}$	No. of plies	Optimum ply thickness (m m) t_1/α_1	t_2/α_2	t_3/α_3	t_4/α_4	Optimum weight $W/\rho A$
0.5	0.0	2	1.33	1.24			2.57
			0.07	0.19			
		4	1.25	0.04	0.04	1.23	2.57
			0.58	41.4	41.4	0.58	
	0.5	2	1.34	1.34			2.68
			9.54	9.54			
		4	0.87	0.48	0.49	0.85	2.68
			6.86	39.43	39.47	6.84	
	1.0	2	1.40	1.41			2.81
			18.18	18.18			
		4	0.84	0.57	0.57	0.84	2.81
			15.5	40.85	40.85	16.5	

TABLE 3.2 CONTINUE

Aspect ratio	$K = \frac{N_y}{N_x}$	No. of plies	Optimum ply thickness (mm) t_1/α_1	t_2/α_2	t_3/α_3	t_4/α_4	Optimum weight $W/\rho A$
	1.5	2	1.46	1.46			2.93
			23.32	23.32			
		4	1.33	0.02	0.04	1.54	2.93
			22.7	52.9	53.0	23.9	
0.8	0.0	2	1.69	1.70			3.39
			37.5	37.45			
		4	1.59	0.10	0.11	1.58	3.39
			37.43	42.47	42.28	37.42	
	0.5	2	1.72	1.72			3.44
			36.78	36.78			
		4	0.98	0.71	0.71	1.13	3.44
			37.71	42.31	42.28	37.9	

TABLE 3.2 CONTINUE

Aspect ratio	$K = \frac{N_y}{N_x}$	No. of plies	Optimum ply thickness (mm) t_1/α_1	t_2/α_2	t_3/α_3	t_4/α_4	Optimum Weight $W/\rho A$
	1.0	2	1.85	1.85			3.70
			38.03	38.03			
		4	1.20	.63	.63	1.24	3.70
			32.25	41.[illegible]6	41.51	37.68	
	1.5	2	1.96	1.96			3.92
			37.33	37.8[illegible]			
		4	1.87	0.15	0.15	1.75	3.92
			37.9	43.0	43.0	37.8	
1.0	0.0	2	2.00	2.00			4.01
			45.0	45.0			
		[illegible]	1.00	1.00	1.00	1.00	4.01
			45.0	45.0	45.0	45.0	
	0.5	2	2.00	2.00			4.01
			45.0	45.0			

TABLE 3.2 CONTINUE

Aspect ratio	$K = \frac{N_y}{N_x}$	No. of plies	Optimum ply thickness (mm) t_1/α_1	t_2/α_2	t_3/α_3	t_4/α_4	Optimum Weight $W/\rho A$
	0.5	4	1.00	1.00	1.00	1.00	4.01
			45.0	45.0	45.0	45.0	
	1.0	2	2.01	2.01			4.02
			45.0	45.0			
		4	1.00	1.00	1.00	1.00	4.02
			45.0	45.0	45.0	45.0	
	1.5	2	2.16	2.16			4.33
			45.0	45.0			
		4	1.08	1.08	1.08	1.08	4.33
			45.0	45.0	45.0	45.0	
1.25	0.0	2	2.28	2.28			4.57
			12.5	52.5			
		4	2.13	0.12	0.14	2.17	4.57
			52.3	48.2	48.0	52.6	

TABLE 3.2 CONTINUE

Aspect ratio	$K = \frac{N_y}{N_x}$	No. of plies	Optimum ply thickness (mm) t_1/α_1	t_2/α_2	t_3/α_3	t_4/α_4	Optimum Weight $W/\rho A$
	0.5	2	2.28	2.28			4.57
			52.8	52.8			
		4	1.74	0.55	0.57	1.70	4.57
			52.7	50.3	50.5	52.5	
	1.0	2	2.29	2.28			4.57
			52.4	52.5			
		4	2.12	1.18	0.21	2.06	4.57
			52.6	48.2	48.2	52.5	
	1.5	2	2.34	2.34			4.68
			52.0	52.0			
		1	1.18	0.89	0.92	1.70	4.69
			52.3	51.7	51.8	52.3	
2.0	0.0	2	2.35	2.35			1.89
			89.9	89.9			
		4	2.15	0.29	1.29	2.16	4.69
			88.9	58.4	58.3	86.9	

TABLE 3.2 CONTINUE

Aspect ratio	$K = \frac{N_y}{N_x}$	No. of plies	Optimum ply thickness t_1/α_1	t_2/α_2	t_3/α_3	t_4/α_4	Optimum Weight $W/\rho A$
2.0	0.5	2	2.45	2.45			4.89
			[illegible]9.9	89.9			
		4	2.39	0.06	0.05	2.39	4.89
			89.8	53.1	53.1	89.8	
	1.0	2	2.46	2.44			4.89
			8[illegible].9	89.9			
		4	2.20	0.23	0.27	2.20	4.89
			[illegible].0	55.2	54.3	89.0	
	1.5	2	[illegible]	2.40			4.98
			7[illegible].0	78.0			
		4	2.[illegible]	0.22	0.23	2.27	4.98
			7[illegible]	55.0	51.9	78.0	

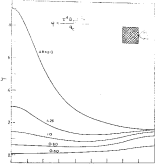

FIG.3.1 MAXIMUM DEFLECTION OF SIMPLY SUPPORTED LAMINATED PLATE VS FIBER ORIENTATION

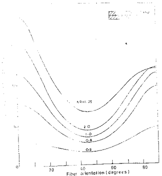

FIG 3.7 VARIATION OF LONGITUDINAL STRESS WITH FIBER ORIENTATION

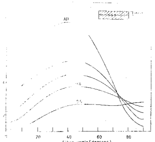

FIG. 3.1 VARIATION OF TRANSVERSE STRESS VS FIBER ORIENTATION

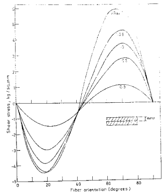

FIG. 3.4 VARIATION OF SHEAR STRESS WITH FIVER ORIENTATION

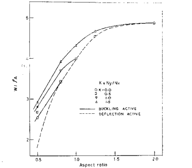

FIG. 3.5 VARIATION OF OPTIMUM WEIGHT WITH ASPECT RATIO

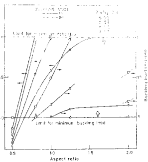

FIG. 3.6 VARIATION OF DEFLECTION AND BUCKLING LOAD WITH ASPECT RATIO FOR OPTIMUM WEIGHT DESIGN

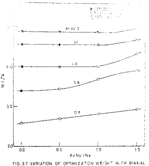

FIG. 3.7 VARIATION OF OPTIMIZATION WEIGHT WITH BIAXIAL LOADING RATIO

CHAPTER 4

OPTIMUM WEIGHT DESIGN OF THE ANTISYMMETRIC LAMINATED COMPOSITE PLATES

4.1 INTRODUCTION

In the previous chapters, a laminated plate with orthotropic laminae is considered for optimal weight design. This orthotropy is achieved by embedding an equal number of fibers at positive and negative orientations with respect to the structural axis, in a matrix or a ply. In this chapter, another class of laminated composite plates is considered, namely, the antisymmetric angle-ply laminates. Coupling between in-plane stretching and twist exists in this case. Also, the coupling stiffness matrix elements, B_{16} and B_{26}, exist in this case. Sharma, et. al. (16) have shown that this coupling can be eliminated completely or partially by choosing the thickness ratios for plies suitably. It is well known that if this coupling is reduced or eliminated, the strength of laminate increases.

Herein, an attempt is made to obtain a minimum weight design for antisymmetric angle-ply laminates subjected to in-plane compressive and transverse loadings. The term antisymmetric is used in the sense that lamina thickness distributions with positive and negative fiber

orientations are located at equal distance from the structural axis. The analysis is quite general and can be applied to any type of antisymmetry.

The following studies are conducted on such plates:

(1) Minimum weight design of antisymmetric angle-ply laminates with variable thickness of plies. The fiber orientations are pre-assigned.

(2) Minimum weight design of antisymmetric angle-ply laminates, treating the thickness of plies and the corresponding fiber orientations as design variables.

4.2 PROBLEM FORMULATION

For these problems the objective function is

$$\min \sum_{i=1}^{N/2} t_i, \quad \text{where, } N = \text{Total no. of plies,} \tag{77}$$

such that

$$-\pi/2 \leqslant \alpha_i \leqslant \pi/2, \qquad i=1, N/2$$

$$t_1 \leqslant t_i \leqslant t_2, \qquad i=1, N/2 \tag{78}$$

$$w \leqslant w^*,$$

$$N_x \geqslant N_x^*,$$

and $$\left(\frac{\sigma_L}{\sigma_{LU}}\right)^2 - \frac{\sigma_L \sigma_T}{\sigma_{LU}^2} + \left(\frac{\sigma_T}{\sigma_{TU}}\right)^2 \left(\frac{\sigma_{LT}}{\sigma_{LTU}}\right)^2 \leqslant 1$$

4.3.1 ANALYSIS

The three governing equations for the displacements u,v and w are :

$$A_{11}\, u_{xx} + A_{66}\, u_{yy} + (A_{12} + A_{66})\, v_{xy} - 3B_{16}\, w_{xxy} - B_{26}\, w_{yyy} = 0 \quad (79)$$

$$(A_{12} + A_{66})\, u_{xy} + A_{66}\, v_{xx} + A_{22} v_{yy} - B_{16}\, w_{xxx} - 3B_{26}\, w_{xyy} = 0 \quad (80)$$

$$D_{11}\, w_{xxxx} + 2\,(D_{12} + 2D_{66})\, w_{xxyy} + D_{22} w_{yyyy} - B_{16}\,(3u_{xxy} + v_{xxx}) - B_{26}(u_{yyy} + 3v_{xyy}) = q \quad (81)$$

(In case of in-plane compressive loading q is equivalent to $(-N_x\, w_{xx} - N_y\, w_{yy})$)

where, the stiffnesses A_{ij} , B_{ij} and D_{ij} are defined in eq. (8). The optimization is performed on a plate which is simply supported on all sides. The boundary conditions are as follows :

Along $x = 0, a$

$v = 0 \qquad M_x = B_{16}\,(u_y + v_x) - D_{11}\, w_{xx} - D_{12}\, w_{yy} = 0$

$u = 0 \qquad N_{xy} = A_{66}(u_y + v_x) - B_{16} w_{xx} - B_{26} w_{yy} = 0$

Along $y = 0, b$,

$$w=0 \quad M_y = B_{26}\,(u_y + v_x) - D_{22}\,w_{yy} - D_{12}\,w_{xx} = 0$$

$$v = 0 \quad N_{xy} = 0 \tag{82}$$

The following displacement functions satisfy the boundary conditions Eqs. (82) exactly,

$$u = \sum\sum \bar{u}_{mn} \sin \frac{m\pi x}{a} \quad \frac{n\pi y}{b}$$

$$v = \sum\sum \bar{v}_{mn} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \tag{83}$$

$$w = \sum\sum \bar{w}_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

Substituting Eq. (83) in Eqs. (79) and (80) we get Eq. (10) as an expression for the buckling load, where T_{ij} are defined by Eq. (11), except T_{13} and T_{26} which are as follows :

$$T_{13} = -\left\{3B_{16}\, m^2 \pi^2 + B_{26}\, n^2 \pi^2 (a/b)^2\right\} n\pi (a/b)$$

$$T_{23} = -\left\{B_{16}\, m^2 \pi^2 + 3B_{26}\, n^2 \pi^2 (a/b)^2\right\} m\pi \tag{84}$$

Deflection under transverse loading is given by Eq. (53), in which, T_{13} and T_{23} are defined in Eq. (84).

4.3.2 CONSTRAINTS AND THEIR GRADIENTS

The following constraints are employed in the interior penalty formulation approach for the optimization:

$$g_i = 1 - t_i/t_2 \qquad i=1, N/2 \tag{85}$$

$$g_{i+N/2} = t_i / t_1 - 1 \qquad i=1, N/2 \tag{86}$$

$$g_{i+N} = 1 - \alpha_i / \pi/2 \qquad i=1, N/2 \tag{87}$$

$$g_{i+3N/2} = 1 - \alpha_i / (-\pi/2) \qquad i=1, N/2 \tag{88}$$

$$g_{2N+1} = 1 - w/w^* \tag{89}$$

$$g_{2N+2} = N_x / N_x^* - 1 \tag{90}$$

and

$$g_{i+2N+2} = 1 - \left(\frac{\sigma_L}{\sigma_{LU}}\right)^2 + \frac{\sigma_L \sigma_T}{\sigma_{LU}} - \left(\frac{\sigma_T}{\sigma_{TU}}\right)^2 - \left(\frac{\sigma_{LT}}{\sigma_{LTU}}\right)^2 \tag{91}$$

The derivatives of the constraints in Eqs. (85) to (88) can easily be obtained and the derivatives of Eqs. (89) to (91) are given by Eqs. (58), (71), Eqs. (12), (72) and Eqs. (60), (61) and (76) respectively.

For problem (1), the constraints (87), (88) are not considered because the fiber orientations are pre-assigned and the thickness of plies are only design variables.

4.4 RESULTS AND DISCUSSIONS

4.4.1 RESULTS

Minimum weight optimization of an antisymmetric laminated composite plate is carried out, for varying number of plies and aspect ratio, for Boron/epoxy composite whose material properties are as given in section (2.4.1).

Numerical results have been computed for the following set of constraints :

Longitudinal tensile strength = 140.57kg/sq.mm.

Transverse tensile strength = 8.43 kg/sq. mm.

Longitudinal compressive strength=-281.13 kg/sq.mm.

Transverse compressive strength =-28.11 kg/sq.mm.

Shear strength =12.65 kg/sq.mm.

Applied uniform transverse loading=0.0005 kg/sq.mm.

Upper limit on maximum deflection=2.00 mm

Minimum buckling load in x-direction=5.00kg/mm.

Minimum thickness of each ply = 0.125 mm.

Maximum thickness of each ply = 2.5 mm

Table 4.1 shows that for a four ply antisymmetric laminate a thickness ratio of 0.414 is attained at optimum design. The fiber orientation of approximately 10° results in minimum weight design for an aspect ratio of 0.5 and k = 0.5.

The variation in ply thickness with increasing number of plies is shown in Table 4.2 and Table 4.3 for an aspect ratio of 1.0 and k=1 and an aspect ratio of 0.5 and k=0.5, respectively.

Table 4.4 gives fiber thickness distribution for different pre-assigned fiber orientations. The maximum fraction of the total thickness is shared by the pair of plies with optimum fiber orientation, while all other plies attained minimum assigned thickness.

Fig. 4.1 presents the variation in ply thickness ratio with increasing number of plies. The thickness ratio of outer plies approaches an asymptotic value. It approaches unity if the constraint on minimum thickness is active for every ply which is possible, if the number of plies is very high. The inner ply thickness ratios are very close to unity and remain almost constant.

The above results are for a pre-assigned fiber orientation. Table 4.5 shows the variation of the optimum fiber orientation and corresponding thickness with aspect ratio and biaxial loading ratio. The optimum weight increases with increasing aspect ratio and biaxial loading ratio. The thickness ratio decreases with increasing biaxial loading ratio. However, the

change in the thickness ratio with biaxial loading ratio diminishes as the aspect ratio approaches unity. The thickness ratio increases with increasing biaxial loading ratio for aspect ratio higher than unity. It seems that the fiber orientation angle ratio increases with increasing aspect ratio. Active constraints for the aspect ratio and the biaxial loading are also presented in this table.

Table 4.4 presents the variation in the ply thickness and the corresponding fiber orientation with increasing number of plies. The fiber orientation of plies near the mid-plane approaches zero with the corresponding thickness approaching the minimum assigned value.

4.4.2 DISCUSSIONS

The optimum thickness ratio to the minimum weight is 0.414 for the four plies antisymmetric laminated plate. This thickness ratio is identical with the one obtained by Sharma, et. al. (16) to remove coupling effects. The thickness ratios for six-ply antisymmetric case are also in accordance with results of reference (16), whose equation is reproduced here

$$R_2 = (R_1 - 1) \pm \sqrt{(2R_1^2 + 2)} \tag{92}$$

where,

$$R_2 = t_2 / t_3$$

and

$$R_1 = t_1 / t_3$$

Taking numerical values from Table [illegible], we get R_2=1.[illegible] and R_1 = 0.7[illegible], which [illegible] (Fig. [illegible]). The minimum weight obtained at the fiber orientations is the same as is obtained in the previous chapter. As the coupling stiffness coefficients are reduced to zero by having a particular thickness distribution, the results given in chapter 2 to get the optimum with respect to fiber orientation apply here as well.

The increase in the number of plies makes a significant change in thickness distribution but the optimum weight remains constant with increasing number of plies. As the number of plies increases, the laminate is identical to the one with orthotropic laminate.

If the ply thickness and the corresponding fiber orientation are taken as the design variables, then the laminate with higher number of plies tries to approach the configuration of four-ply antisymmetric laminate.

The variation of the optimum weight with the

aspect ratio and the biaxial loading is according to the explanation given in chapter 3.

4.5. CONCLUSIONS

The following conclusions are drawn from the results obtained :

(1) A particular thickness distribution results in the optimum weight design for the pre-assigned fiber orientation for the alternate plies.

(2) When the thickness and the fiber orientation are taken as the design variables, it appears that in addition to the thickness ratio approaching an optimum value, the ratio of fiber orientations also approach an optimum.

(3) The general conclusions drawn in chapters 2 and 3 are also applicable here as discussed in section 4.4.2.

TABLE 4.1 VARIATION IN THE OPTIMUM WEIGHT WITH CHANGE IN PREASSIGNED ALTERNATE FIBER ORIENTATION

Total number of plies = 4

Aspect ratio = 0.5

$K = N_y / N_x$ = 0.5

Alternate fiber Orientation	Optimum Values t_1	Optimum Values t_2	Optimum Weight $(W/\rho L)$	t_1/t_2
± 30°	0.4103	0.9885	2.7976	0.415
± 45°	0.4420	1.0[illegible]6	3.0172	0.414
± 10°	0.3928	0.9488	2.6832	0.414

$K = N_y / N_x \quad = 0.0$

Alternate fiber orientation = $\pm$ 30

Total no. of plies	Optimum thickness										Optimum Weight W/ρA
	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	
4	0.6087	1.4692									4.1558
6	0.4667	0.9508	0.6603								4.1556
8	0.3824	0.6806	0.4554	0.5595							4.1558
12	0.2779	0.4263	0.3017	0.3886	0.3275	0.3557					4.1554
16	0.2224	0.3075	0.2299	0.2906	0.2391	0.2756	0.2501	0.2626			4.1588
20	0.1829	0.2395	0.1875	0.2307	0.1919	0.2228	0.1968	0.2151	0.2025	0.2084	4.1558

Aspect ratio = 0.5

$K = N_y / N_x$ = 0.5

Alternate fiber orientation = $\pm$ 10

Total No.of plies	Optimum thickness										Optimum Weight W/ A
	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	
4	0.3928	0.9488									2.6832
6	0.3048	0.6184	0.4183								2.6830
8	0.2500	0.4459	0.2936	0.3522							2.6832
12	0.1840	0.2806	0.1956	0.2491	0.2076	0.2247					2.6832
16	0.1468	0.2132	0.1522	0.1836	0.1552	0.1695	0.1597	0.1623			2.6850
20	0.1271	0.1924	0.1269	0.1374	0.1257	0.1270	0.1261	0.1273	0.1264	0.1252	2.6830

Total number of plies = 20

Aspect ratio = 1.0

$K = N_y/N_x$ = 0.0

Alternate fiber orientation	Optimum thickness										Optimum Weight W/ A
	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	
±30	0.1829	0.2395	0.1873	0.2307	0.1919	0.2228	0.1968	0.2151	0.2025	0.2084	4.1558
±45	0.1776	0.2323	0.1815	0.2226	0.1842	0.2143	0.1892	0.2072	0.1954	0.2006	4.0098
±45	0.4225	0.6233									4.0918
±30			0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1251	
±30	0.1250	0.1256	0.1250	0.1252	0.1250	0.1251	0.1250	0.1250			4.0702
[illegible]									0.2802	0.7540	

Number of plies = 4

Aspect ratio	$K = \frac{N_y}{N_x}$	Optimum values				Optimum Weight $W/\rho A$	Active Constraint	$\frac{t_1}{t_2}$	$-\frac{\alpha_1}{\alpha_2}$
		t_1	t_2	α_1	α_2				
0.5	0.0	.6423	.6420	0.01	-0.04	2.57	Buckling	1.0	.25
	0.5	.5591	.7821	7.56	-1544	2.68	Buckling	.714	.489
	1.0	.4983	.9081	15.44	-25.54	2.81	Buckling	.548	.605
0.8	0.0	.4971	1.1991	37.2	-37.42	3.39	deflection	.414	.994
	0.5	.5043	1.2161	38.01	-38.16	3.44	Buckling	.414	.996
1.0	0.0	.5867	1.4182	45.10	-45.14	4.01	deflection	.414	.999
	0.5	.5873	1.4176	44.90	-4492	4.01	deflection	.414	.999
	1.0	.5885	1.4214	44.93	-44.87	4.02	Buckling	.414	1.001
2.0	0.0	2.2313	0.2165	85.99	-58.80	4.89	deflection	10.30	1.53
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4.89	deflection	18.59	1.497

Aspect ratio = 1.0

$K = N_y/N_x$ = 0.0

Total Number of Plies	Optimum Values						Optimum Weight $W/\rho A$
	t_1/α_1	t_2/α_2	t_3/α_3	t_4/α_4	t_5/α_5	t_6/α_6	
4	.5867 45.10	1.4182 -45.14					4.01
6	.5856 44.93	1.2945 -44.98	.1250 -837				4.01
8	.5688 44.23	.5912 -45.04	.5301 -41.56	.3204 1.57			4.02
10	.5123 42.72	.6507 -44.75	.3029 -1658	.2887 -2.10	.2869 -034		4.08
12	.5374 -45.25	.3720 46.09	.1506 -0.86	.1528 5.82	.1490 2.06	.1520 0.10	4.03

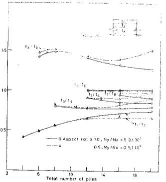

FIG. 4.1 THICKNESS RATIO VS. TOTAL NUMBER OF PLIES

CHAPTER 5

CONCLUSION

The key to the effective design of composite structures is the proper use of the optimization techniques for more efficient utilization of the material. An unfortunate circumstance which restricts the maximum use of all the design flexibilities provided by composites is the intrinsic complexities involved at micro-mechanical and macro-mechanical levels of the analysis of such materials. However, lot of theoretical advancements have been made and the usage of optimization techniques is gaining momentum.

In order to comprehend and appreciate the vital role which the application of optimization techniques can play, it is worthwhile to point out, here, that a reduction by one-tenth of a millimeter of thickness, for a plate of size (50 cm x 50cm), results in material cost-savings of approximately $ 40 , Boron/epoxy composite cost projection, at present. This does not include the operational cost-savings, which will be tremendous in the case of aerospace applications.

In this thesis, the optimization studies have been carried out, incorporating a set of design variables to achieve more flexibility in the design, to get a

minimum weight design of a laminated composite plate. Particular cases of laminates with especially orthotropic laminae and antisymmetric angle-ply laminates were considered. Maximum flexibility in choosing the design variables was utilized while restricting the analysis to analytical frame-work to avoid numerical techniques, which, besides being approximate in nature are also time consuming. Simply supported boundary conditions which are amenable to closed form solution have been used throughout the study.

In the following sections, the conclusions drawn at the end of each chapter is summarised.

The constrained optimization problem is transformed into a series of unconstrained optimization problems, using interior penalty function approach. It should be noted that the fiber orientations (design variables) enter the transformed function through the constraints. Objective function does not contain the fiber orientation as variables. For higher values of the penalty parameter, none of the constraints become active, (transformed objective function has the minimum away from the constrai surfaces) or in other words, behavioral and side constrai contribute almost equally to the objective function. This results in the fiber orientation achieving an optimum value, with every constraint contributing almost equally

to it. During sequential optimization, when the penalty parameter is reduced to a lower value, the active constrain contribute dominantly to the objective function. Due to this, the fiber orientation of the inner plies, whose gradients are low compared with other variables, may not reach the optimum value. This does not affect the optimum weight appreciably.

The total computational time taken by the algorithm is dependent on the initial point, the final optimum value and other factors. In view of this, any comparison of total computational time is not attempted. Each stress and deflection analysis takes about 0.33 second and, if analytical derivatives are calculated, an additional time of about 0.07 second is taken. For calculating the numerical derivatives, the algorithm takes about 0.65 second. Hence, analytical derivative calculation results in less CPU (Central Processing Unit) time.

The following general conclusions can be drawn from the study conducted :

(1) Maximum buckling load is obtained when the total thickness of the plate assumes the optimum fiber orientation for a given aspect ratio and biaxial loading ratio.

(2) The designer of the composite plate can work with relaxed manufacturing tolerances, as far as plies near the mid-plane are concerned.

(3) The weight per unit area of the plate increases with increasing aspect ratio. However, it seems to approach an asymptotoc value with aspect ratio.

(4) Stresses never become active at optimum with the low deflection requirement on the laminate design. These constraints can be deleted to improve the efficiency of the algorithm.

(5) The optimum for an antisymmetric angle-ply laminate is obtained when the ply thicknesses achieve particular ratios.

EXTENSION OF THE PRESENT WORKS

The logical extension of the present work can be stated as follows :

(1) A general antisymmetric laminated ply can be considered with various boundary conditions.

(2) Volume fraction of the fiber and the matrix of each ply can be incorporated as a design variable to get an optimum weight structure.

(3) Optimization of hybrid laminates.

REFERENCES

(1) Mroz, Z., "Optimal design of structures of composite materials", International J of solids and structures, Vol. 6, No. 7, 1970, pp. 859 - 870.

(2) Bryzgalin, G.I., "On some optimal design criteria of inhomogeneous anisotropic bodies", J of Applied Mathematics and Mechanics, Vol. 36, No. 4, 1972, pp. 716 - 723.

(3) Love, P.G. and Melchers, R.E., "On the theory of optimal constant thickness fiber reinforced plates", International J of Mechanical Sciences, Vol. 15, 1973, pp. 311 - 324.

(4) Khot, N.S., and Venkayya, V.B., Johnson, C.D. and Tischler, V.A., "Optimization of fiber reinforced composite structures", International J of solids and structures, Vol. 9, 1973, pp. 1225 - 1236.

(5) Lai, Y.S. and Achenbach, J.D., "Optimal design of layered structures under dynamic loading", Computer and structure, Vol. 3, 1973, pp. 559 - 572.

(6) Schmit (Jr.), L.A. and Farshi, B., "Optimum laminate design for strength and stiffness", International J for Numerical Engineering, Vol. 7, No. 4, 1973, pp. 519 - 536.

(7) Isaakson, G., Pardo, H. and Lesko, E., "A program for optimum design of metallic and composite structures subject to strength and deflection constraints", Proc. structures, structural dynamics and material conference, San Diego, California, March 23-24, 1977.

(8) Khot, N.S., "Computer programme (OPTCOMP) for optimization of composite structures for minimum weight design", U.S. Airforce Dynamic Laboratory, Tech. report AFFDL-TR-76- 149, 1977, pp. 58.

(9) McKeown, J.J., "Optimal composite structure by deflection variable programming", Computer Methods in Applied Mechanics and Engineering, Vol. 12, 1977, pp. 155-179.

(10) McKeown, J.J. "A New approach to the optimisation multi laminar composite sheets", Proc. International council of Aeronautical Science conference 11th, Lisbon, Potugal, Sept. 10-16, 1978, Vol. 1.

(11) McKeown, J.J., "A quasi-linear programming algorithm for optimizing fiber - reinforced structure of fixed stiffness", Computer Methods in Applied Mechanics and Engineering, Vol. 6, 1975, pp. 123-154.

(12) Chao, C.C., Koh, S.L. and Sun, C.T., "Optimization of buckling and yield strengths of laminated composites",AIAA Journal, Vol. 13, No.9, 1975, pp. 1131 - 1132.

(13) Schmit (Jr.), L.A. and Farshi, B, "Optimum design of laminated fiber composite plates", International J for Numerical Methods in Engineering, Vol. 11, No. 4, 1977, pp. 623-640.

(14) Hayashi, T. "Optimization for elastic buckling strength of fiber reinforced composite structures-columns, plates and cylinders", Proceedings of symposium, Kyoto, Society of Material Science Japan, 1974, pp. 399-405.

(15) Hirano, Y., "Optimum design of laminated plates under axial compression, AIAA Journal, Vol. 17, 1979, pp. 1017-1019.

(16) Sharma, S., Iyengar, N.G.R. and Murthy P.N., "Some comments on coupling of angle ply laminates", J. structural Mechanics, Vol. 7, 1979, pp. 473-482.

(17) Broutman, L.J., and Krock, R.H., Ed. "Composite Materials" Vol.3, Academic Press. 1974.

(18) Iyengar, N.G.R., and Gupta, S.K., "Programming methods in structural design" Affiliated East-West Press Private Limited, Delhi, 1980.

(19) Jones, R.M., " Mechanics of composite materials",
Scripta Book Company, 1975.

(20) Calcote, L.R., "The analysis of laminated composite structures", Van Nostrand Reinhold Company, New York, 1969.

APPENDIX 1

The gradients of the stiffness matrices are obtained in the following manner.

$$\frac{\partial A_{ij}}{\partial v_l} = \sum_{k=1}^{N} \left\{ \frac{\partial \bar{Q}_{ij}^{(k)}}{\partial \alpha_m} (z_k - z_{k-1}) + \bar{Q}_{ij}^{(k)} \left(\frac{\partial z_k}{\partial t_n} - \frac{\partial z_{k-1}}{\partial t_n} \right) \right\}$$

$$\frac{\partial B_{ij}}{\partial v_l} = \sum_{k=1}^{N} \left\{ \frac{1}{2} \frac{\partial \bar{Q}_{ij}^{(k)}}{\partial \alpha_m} (z_k^2 - z_{k-1}^2) \right.$$

$$\left. + \bar{Q}_{ij}^{(k)} \left(z_k \frac{\partial z_k}{\partial t_n} - z_{k-1} \frac{\partial z_{k-1}}{\partial t_n} \right) \right\}$$

$$\frac{\partial D_{ij}}{\partial v_l} = \sum_{k=1}^{N} \left\{ \frac{1}{3} \frac{\partial \bar{Q}_{ij}^{(k)}}{\partial \alpha_m} (z_k^3 - z_{k-1}^3) \right.$$

$$\left. + \bar{Q}_{ij}^{(k)} \left(z_k^2 \frac{\partial z_k}{\partial t_n} - z_{k-1}^2 \frac{\partial z_{k-1}}{\partial t_n} \right) \right\}$$

where

$v_l = t_n$ for $l=1, N'/2$

$n=1, N$

$v_l = \alpha_m$ for $l=N'/2+1, N'$

$n=N+1, 2N$

and

$$\frac{\partial \bar{Q}_{ij}^{(k)}}{\partial \alpha_m} = 0 \qquad \text{for } k \neq m$$

and

$$\frac{\partial \bar{Q}_{11}^{(k)}}{\partial \alpha_m} = -2U_2 \sin (2 \alpha_m) - 4 U_3 \sin (4 \alpha_m)$$

$$\frac{\partial \bar{Q}_{12}^{(k)}}{\partial \alpha_m} = 4 U_3 \sin (4 \alpha_m)$$

$$\frac{\partial \bar{Q}_{22}^{(k)}}{\partial \alpha_m} = 2 U_2 \sin (2 \alpha_m) - 4 U_3 \sin (4 \alpha_m)$$

$$\frac{\partial \bar{Q}_{16}^{(k)}}{\partial \alpha_m} = U_2 \cos (2 \alpha_m) + 4 U_3 \cos (4 \alpha_m)$$

$$\frac{\partial \bar{Q}_{26}^{(k)}}{\partial \alpha_m} = U_2 \cos (2 \alpha_m) - 4 U_3 \cos (4 \alpha_m)$$

$$\frac{\partial \bar{Q}_{66}^{(k)}}{\partial \alpha_m} = 4 U_3 \sin (4 \alpha_m)$$

for $k = m$

where

$$U_2 = \frac{Q_{11} - Q_{22}}{2}$$

$$U_3 = \frac{Q_{11} + Q_{22} - Q_{12} - 4Q_{66}}{8}$$

and

$$\frac{\partial z_k}{\partial t_n} = -\frac{1}{2} \qquad k < n$$

and $k \neq MJ$

$$= \frac{1}{2} \qquad k > n$$

$$= 0 \qquad k = MJ$$

where

$$
\begin{aligned}
z_k &= -\frac{1}{2} \sum_{i=1}^{N} t_i && k = 0 \\
&= -\frac{1}{2} \sum_{i=1}^{N} t_i - \sum_{i=1}^{k} t_i && k = 1,\ NJ-1 \\
&= -\frac{1}{2} \sum_{i=1}^{N} - \sum_{i=k}^{N} t_i && k = NJ+1, N \\
&= \frac{1}{2} \sum_{i=1}^{N} t_i && k = N+1 \\
&= 0 && k = NJ
\end{aligned}
$$

(See Fig. 2.1)